ON TOPOLOGY OF TORIC SPACES ARISING FROM 2-TRUNCATED CUBES

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To any convex simple n-dimensional polytope P with m facets one can associate its moment-angle manifold $\mathcal{Z}_P$. This construction appeared firstly in the work of M.Davis and T.Januszkiewicz as a generalization of the notion of a topological toric manifold. Moment-angle manifolds and more general polyhedral products were then studied intensively and became one of the main objects of study in toric topology. V.Buchstaber and T.Panov proved that $\mathcal{Z}_P$ is a smooth $(m+n)$-dimensional closed 2-connected manifold with a compact torus T^m action, whose orbit space is homeomorphic to the polytope P itself. They also proved that its cohomology algebra is isomorphic to the Tor-algebra $\mathrm{Tor}_{k[v_1,\ldots,v_m]}(k[P],k)$ of P over a polynomial algebra, where k is a commutative ring with a unit and $k[P]$ is a Stanley-Reisner ring. The latter is isomorphic to the T^m-equivariant cohomology ring $H^*_{T^m}(\mathcal{Z}_P)$. Studying Massey products in these Tor-algebras is a graded version of a classical problem on homology of local rings known since 1960s. The topology of $\mathcal{Z}_P$ is governed by the face lattice of P and can be very complicated.

T.Panov and N.Ray proved that quasitoric manifolds are formal spaces; projective toric manifolds are Kähler and thus formal due to P.Deligne, P.Griffiths, J.Morgan and D.Sullivan. It is well known that a nontrivial higher order Massey operation in cohomology is an obstruction to formality. I.Baskakov found a class of triangulated spheres K for which $\mathcal{Z}_K$ are non-formal manifolds, having a nontrivial triple Massey product of 3-dimensional cohomology classes. Later on, this case was totally described in terms of combinatorics of K by G.Denham and A.Suciu.

In this talk we are going to discuss higher Massey products in cohomology and formality of moment-angle manifolds $\mathcal{Z}_P$ when P is a 2-truncated cube, that is a consecutive cut of only codimension 2 faces starting with a cube. We introduce a family of n-dimensional flag nestohedra P which starts with a simple 3-polytope combinatorially dual to the Baskakov 2-sphere, such that there is a nontrivial n-fold Massey product in cohomology of the moment-angle manifold $\mathcal{Z}_P$ for any $n \geq 3$. V.Buchstaber and V.Volodin proved that any flag nestohedron can be realized as a 2-truncated cube; we present our family of flag nestohedra as 2-truncated n-cubes for $n \geq 3$. Then we discuss some results and problems concerning nontrivial triple Massey products and formality for $\mathcal{Z}_P$ when P is a graph-associahedron of M.Carr and S.Devadoss or a generalized associahedron of S.Fomin and A.Zelevinsky, based on the properties of these polytopes arising in representation theory, cluster algebras and convex geometry.

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