On the Morel-Voevodsky K-theory Representability Theorem

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The Morel-Voevodsky K-theory representability theorem [MV, Prop.3.9, p.139; Th.3.13, p.140] manifestly represents the usefulness of the Morel-Voevodsky \mathbb{A}^1 -homotopy theory. However, Morel-Voevodsky [MV] did not supply enough explanation for the proof, apparently even for experts...

In [M, Th.3.3, p.90; Th.3.4, p.91], I presented a somewhat detailed proof for the Morel-Voevodsky K-theory representability theorem, hoping to be understandable even for non experts like the transformation group theorists.

While my presentation [M] was, just like the original Morel-Voevodsky setting [MV], still concerned the representability in the motivic unstable homotopy category, Cisinski [C] went further to prove the K-theory representability in the motivic stable homotopy category.

Cisinski [C] further applied it, together with results obtained by Ayoub's two Asterisque volumes [A], to prove his cdh descent theorem for the homotopy invariant K-theory [C, Theorem 3.9, p.18].

In this talk, I shall review these developments.

References

- [A] Ayoub, Asterisque, 314, 315 (2008), Societe Mathematique de France.
- [C] Cisinski, Ann. of Math. (2) 177 (2013), no. 2, 425–448.
- [M] Minami, RIMS Kokuroku Bessatsu B39 (2013), 063–107.

[MV] Morel-Voevodsky, Publications mathematiques de l'I.H.E.S. 90 (1999) 45-143.