Smooth actions on complex projective spaces

Krzysztof M. Pawałowski
Adam Mickiewicz University · Poznań · Poland

43rd Symposium on Transformation Groups
November 17–19, 2016 · Himeji · Japan

The goal

The goal

The goal of the lecture is to present constructions of smooth actions of compact Lie groups G

The goal

The goal of the lecture is to present constructions of smooth actions of compact Lie groups G on complex projective spaces $\mathbb{C}P^n$,

The goal

The goal of the lecture is to present constructions of smooth actions of compact Lie groups G on complex projective spaces $\mathbb{C}P^n$, such that the manifold

$$F(G \circlearrowright \mathbb{C}P^n) = \{x \in \mathbb{C}P^n \colon g \cdot x = x \text{ for all } g \in G\}$$

The goal

The goal of the lecture is to present constructions of smooth actions of compact Lie groups G on complex projective spaces $\mathbb{C}P^n$, such that the manifold

$$F(G \circlearrowright \mathbb{C}P^n) = \{x \in \mathbb{C}P^n \colon g \cdot x = x \text{ for all } g \in G\}$$

consisting of points fixed under the action of G on $\mathbb{C}P^n$

The goal

The goal of the lecture is to present constructions of smooth actions of compact Lie groups G on complex projective spaces $\mathbb{C}P^n$, such that the manifold

$$F(G \circlearrowright \mathbb{C}P^n) = \{x \in \mathbb{C}P^n \colon g \cdot x = x \text{ for all } g \in G\}$$

consisting of points fixed under the action of G on $\mathbb{C}P^n$ contains a connected component M admitting some geometric structure while missing another one.

The goal

The goal of the lecture is to present constructions of smooth actions of compact Lie groups G on complex projective spaces $\mathbb{C}P^n$, such that the manifold

$$F(G \circlearrowright \mathbb{C}P^n) = \{x \in \mathbb{C}P^n \colon g \cdot x = x \text{ for all } g \in G\}$$

consisting of points fixed under the action of G on $\mathbb{C}P^n$ contains a connected component M admitting some geometric structure while missing another one. We shall see how to construct smooth actions of G on $\mathbb{C}P^n$,

The goal

The goal of the lecture is to present constructions of smooth actions of compact Lie groups G on complex projective spaces $\mathbb{C}P^n$, such that the manifold

$$F(G \circlearrowright \mathbb{C}P^n) = \{x \in \mathbb{C}P^n \colon g \cdot x = x \text{ for all } g \in G\}$$

consisting of points fixed under the action of G on $\mathbb{C}P^n$ contains a connected component M admitting some geometric structure while missing another one. We shall see how to construct smooth actions of G on $\mathbb{C}P^n$, in such a way that

The goal

The goal of the lecture is to present constructions of smooth actions of compact Lie groups G on complex projective spaces $\mathbb{C}P^n$, such that the manifold

$$F(G \circlearrowright \mathbb{C}P^n) = \{x \in \mathbb{C}P^n \colon g \cdot x = x \text{ for all } g \in G\}$$

consisting of points fixed under the action of G on $\mathbb{C}P^n$ contains a connected component M admitting some geometric structure while missing another one. We shall see how to construct smooth actions of G on $\mathbb{C}P^n$, in such a way that

- M is stable almost complex and M is not almost complex,

The goal

The goal of the lecture is to present constructions of smooth actions of compact Lie groups G on complex projective spaces $\mathbb{C}P^n$, such that the manifold

$$F(G \circlearrowright \mathbb{C}P^n) = \{x \in \mathbb{C}P^n \colon g \cdot x = x \text{ for all } g \in G\}$$

consisting of points fixed under the action of G on $\mathbb{C}P^n$ contains a connected component M admitting some geometric structure while missing another one. We shall see how to construct smooth actions of G on $\mathbb{C}P^n$, in such a way that

- M is stable almost complex and M is not almost complex,
- M is almost complex and M is not homotopically symplectic,

The goal

The goal of the lecture is to present constructions of smooth actions of compact Lie groups G on complex projective spaces $\mathbb{C}P^n$, such that the manifold

$$F(G \circlearrowright \mathbb{C}P^n) = \{x \in \mathbb{C}P^n \colon g \cdot x = x \text{ for all } g \in G\}$$

consisting of points fixed under the action of G on $\mathbb{C}P^n$ contains a connected component M admitting some geometric structure while missing another one. We shall see how to construct smooth actions of G on $\mathbb{C}P^n$, in such a way that

- M is stable almost complex and M is not almost complex,
- M is almost complex and M is not homotopically symplectic,
- M is homotopically symplectic and M is not symplectic.

The goal

The goal of the lecture is to present constructions of smooth actions of compact Lie groups G on complex projective spaces $\mathbb{C}P^n$, such that the manifold

$$F(G \circlearrowright \mathbb{C}P^n) = \{x \in \mathbb{C}P^n \colon g \cdot x = x \text{ for all } g \in G\}$$

consisting of points fixed under the action of G on $\mathbb{C}P^n$ contains a connected component M admitting some geometric structure while missing another one. We shall see how to construct smooth actions of G on $\mathbb{C}P^n$, in such a way that

- M is stable almost complex and M is not almost complex,
- M is almost complex and M is not homotopically symplectic,
- M is homotopically symplectic and M is not symplectic.

Almost complex actions,

The goal

The goal of the lecture is to present constructions of smooth actions of compact Lie groups G on complex projective spaces $\mathbb{C}P^n$, such that the manifold

$$F(G \circlearrowright \mathbb{C}P^n) = \{x \in \mathbb{C}P^n \colon g \cdot x = x \text{ for all } g \in G\}$$

consisting of points fixed under the action of G on $\mathbb{C}P^n$ contains a connected component M admitting some geometric structure while missing another one. We shall see how to construct smooth actions of G on $\mathbb{C}P^n$, in such a way that

- M is stable almost complex and M is not almost complex,
- M is almost complex and M is not homotopically symplectic,
- M is homotopically symplectic and M is not symplectic.

Almost complex actions, homotopically symplectic actions,

The goal

The goal of the lecture is to present constructions of smooth actions of compact Lie groups G on complex projective spaces $\mathbb{C}P^n$, such that the manifold

$$F(G \circlearrowright \mathbb{C}P^n) = \{x \in \mathbb{C}P^n \colon g \cdot x = x \text{ for all } g \in G\}$$

consisting of points fixed under the action of G on $\mathbb{C}P^n$ contains a connected component M admitting some geometric structure while missing another one. We shall see how to construct smooth actions of G on $\mathbb{C}P^n$, in such a way that

- M is stable almost complex and M is not almost complex,
- M is almost complex and M is not homotopically symplectic,
- M is homotopically symplectic and M is not symplectic.

Almost complex actions, homotopically symplectic actions, and symplectic actions

The goal

The goal of the lecture is to present constructions of smooth actions of compact Lie groups G on complex projective spaces $\mathbb{C}P^n$, such that the manifold

$$F(G \circlearrowright \mathbb{C}P^n) = \{x \in \mathbb{C}P^n \colon g \cdot x = x \text{ for all } g \in G\}$$

consisting of points fixed under the action of G on $\mathbb{C}P^n$ contains a connected component M admitting some geometric structure while missing another one. We shall see how to construct smooth actions of G on $\mathbb{C}P^n$, in such a way that

- M is stable almost complex and M is not almost complex,
- M is almost complex and M is not homotopically symplectic,
- M is homotopically symplectic and M is not symplectic.

Almost complex actions, homotopically symplectic actions, and symplectic actions of compact Lie groups G on $\mathbb{C}P^n$

The goal

The goal of the lecture is to present constructions of smooth actions of compact Lie groups G on complex projective spaces $\mathbb{C}P^n$, such that the manifold

$$F(G \circlearrowright \mathbb{C}P^n) = \{x \in \mathbb{C}P^n \colon g \cdot x = x \text{ for all } g \in G\}$$

consisting of points fixed under the action of G on $\mathbb{C}P^n$ contains a connected component M admitting some geometric structure while missing another one. We shall see how to construct smooth actions of G on $\mathbb{C}P^n$, in such a way that

- M is stable almost complex and M is not almost complex,
- M is almost complex and M is not homotopically symplectic,
- M is homotopically symplectic and M is not symplectic.

Almost complex actions, homotopically symplectic actions, and symplectic actions of compact Lie groups G on $\mathbb{C}P^n$ form three different classes of transformation groups.

Almost complex and stable almost complex manifolds

By an *almost complex* manifold (M^{2n}, J)

Almost complex and stable almost complex manifolds

By an *almost complex* manifold (M^{2n}, J) we mean a smooth manifold M^{2n} with a vector bundle map $J\colon T(M) \to T(M)$ over id_M

Almost complex and stable almost complex manifolds

By an *almost complex* manifold (M^{2n}, J) we mean a smooth manifold M^{2n} with a vector bundle map $J\colon T(M) \to T(M)$ over id_M such that $J^2 = -id_{T(M)}$.

Almost complex and stable almost complex manifolds

By an *almost complex* manifold (M^{2n}, J) we mean a smooth manifold M^{2n} with a vector bundle map $J\colon T(M) \to T(M)$ over id_M such that $J^2 = -id_{T(M)}$. In other words, the tangent bundle $T(M)$ admists the structure of a complex vector bundle.

Almost complex and stable almost complex manifolds

By an *almost complex* manifold (M^{2n}, J) we mean a smooth manifold M^{2n} with a vector bundle map $J\colon T(M) \to T(M)$ over id_M such that $J^2 = -id_{T(M)}$. In other words, the tangent bundle $T(M)$ admits the structure of a complex vector bundle.

Existence of an almost complex structure J on M is equivalent to existence of a differential 2-form ω on M

Almost complex and stable almost complex manifolds

By an *almost complex* manifold (M^{2n}, J) we mean a smooth manifold M^{2n} with a vector bundle map $J\colon T(M) \to T(M)$ over id_M such that $J^2 = -id_{T(M)}$. In other words, the tangent bundle $T(M)$ admists the structure of a complex vector bundle.

Existence of an almost complex structure J on M is equivalent to existence of a differential 2-form ω on M which is non-degenerate at each point $x \in M$,

Almost complex and stable almost complex manifolds

By an *almost complex* manifold (M^{2n}, J) we mean a smooth manifold M^{2n} with a vector bundle map $J\colon T(M) \to T(M)$ over id_M such that $J^2 = -id_{T(M)}$. In other words, the tangent bundle $T(M)$ admists the structure of a complex vector bundle.

Existence of an almost complex structure J on M is equivalent to existence of a differential 2-form ω on M which is non-degenerate at each point $x \in M$, i.e., for any non-zero vector $v \in T_x(M)$, the 1-form $\{\lambda_x\colon T_x(M) \to \mathbb{R}\}_{x \in M}$ given by $\lambda_x(w) = \omega_x(v, w)$ do not vanish.

Almost complex and stable almost complex manifolds

By an *almost complex* manifold (M^{2n}, J) we mean a smooth manifold M^{2n} with a vector bundle map $J\colon T(M) \to T(M)$ over id_M such that $J^2 = -id_{T(M)}$. In other words, the tangent bundle $T(M)$ admists the structure of a complex vector bundle.

Existence of an almost complex structure J on M is equivalent to existence of a differential 2-form ω on M which is non-degenerate at each point $x \in M$, i.e., for any non-zero vector $v \in T_x(M)$, the 1-form $\{\lambda_x\colon T_x(M) \to \mathbb{R}\}_{x \in M}$ given by $\lambda_x(w) = \omega_x(v, w)$ do not vanish.

By a *stable almost complex* manifold (M, f)

Almost complex and stable almost complex manifolds

By an *almost complex* manifold (M^{2n}, J) we mean a smooth manifold M^{2n} with a vector bundle map $J\colon T(M) \to T(M)$ over id_M such that $J^2 = -id_{T(M)}$. In other words, the tangent bundle $T(M)$ admists the structure of a complex vector bundle.

Existence of an almost complex structure J on M is equivalent to existence of a differential 2-form ω on M which is non-degenerate at each point $x \in M$, i.e., for any non-zero vector $v \in T_x(M)$, the 1-form $\{\lambda_x\colon T_x(M) \to \mathbb{R}\}_{x \in M}$ given by $\lambda_x(w) = \omega_x(v, w)$ do not vanish.

By a *stable almost complex* manifold (M, f) we mean a smooth manifold M with a smooth embedding $f\colon M \to E$ into some Euclidean space E

Almost complex and stable almost complex manifolds

By an *almost complex* manifold (M^{2n}, J) we mean a smooth manifold M^{2n} with a vector bundle map $J\colon T(M) \to T(M)$ over id_M such that $J^2 = -id_{T(M)}$. In other words, the tangent bundle $T(M)$ admits the structure of a complex vector bundle.

Existence of an almost complex structure J on M is equivalent to existence of a differential 2-form ω on M which is non-degenerate at each point $x \in M$, i.e., for any non-zero vector $v \in T_x(M)$, the 1-form $\{\lambda_x\colon T_x(M) \to \mathbb{R}\}_{x\in M}$ given by $\lambda_x(w) = \omega_x(v, w)$ do not vanish.

By a *stable almost complex* manifold (M, f) we mean a smooth manifold M with a smooth embedding $f\colon M \to E$ into some Euclidean space E such that the normal bundle of the embedding f admits a complex structure

Almost complex and stable almost complex manifolds

By an *almost complex* manifold (M^{2n}, J) we mean a smooth manifold M^{2n} with a vector bundle map $J\colon T(M) \to T(M)$ over id_M such that $J^2 = -id_{T(M)}$. In other words, the tangent bundle $T(M)$ admits the structure of a complex vector bundle.

Existence of an almost complex structure J on M is equivalent to existence of a differential 2-form ω on M which is non-degenerate at each point $x \in M$, i.e., for any non-zero vector $v \in T_x(M)$, the 1-form $\{\lambda_x\colon T_x(M) \to \mathbb{R}\}_{x \in M}$ given by $\lambda_x(w) = \omega_x(v, w)$ do not vanish.

By a *stable almost complex* manifold (M, f) we mean a smooth manifold M with a smooth embedding $f\colon M \to E$ into some Euclidean space E such that the normal bundle of the embedding f admits a complex structure or, equivalently, the tangent bundle $T(M)$ to M admits the structure of a complex vector bundle,

Almost complex and stable almost complex manifolds

By an *almost complex* manifold (M^{2n}, J) we mean a smooth manifold M^{2n} with a vector bundle map $J\colon T(M) \to T(M)$ over id_M such that $J^2 = -id_{T(M)}$. In other words, the tangent bundle $T(M)$ admits the structure of a complex vector bundle.

Existence of an almost complex structure J on M is equivalent to existence of a differential 2-form ω on M which is non-degenerate at each point $x \in M$, i.e., for any non-zero vector $v \in T_x(M)$, the 1-form $\{\lambda_x\colon T_x(M) \to \mathbb{R}\}_{x \in M}$ given by $\lambda_x(w) = \omega_x(v, w)$ do not vanish.

By a *stable almost complex* manifold (M, f) we mean a smooth manifold M with a smooth embedding $f\colon M \to E$ into some Euclidean space E such that the normal bundle of the embedding f admits a complex structure or, equivalently, the tangent bundle $T(M)$ to M admits the structure of a complex vector bundle, after adding the product bundle $M \times \mathbb{R}^k$ over M for an integer $k \geqslant 0$.

Almost complex and stable almost complex manifolds

By an *almost complex* manifold (M^{2n}, J) we mean a smooth manifold M^{2n} with a vector bundle map $J\colon T(M) \to T(M)$ over id_M such that $J^2 = -id_{T(M)}$. In other words, the tangent bundle $T(M)$ admits the structure of a complex vector bundle.

Existence of an almost complex structure J on M is equivalent to existence of a differential 2-form ω on M which is non-degenerate at each point $x \in M$, i.e., for any non-zero vector $v \in T_x(M)$, the 1-form $\{\lambda_x\colon T_x(M) \to \mathbb{R}\}_{x \in M}$ given by $\lambda_x(w) = \omega_x(v, w)$ do not vanish.

By a *stable almost complex* manifold (M, f) we mean a smooth manifold M with a smooth embedding $f\colon M \to E$ into some Euclidean space E such that the normal bundle of the embedding f admits a complex structure or, equivalently, the tangent bundle $T(M)$ to M admits the structure of a complex vector bundle, after adding the product bundle $M \times \mathbb{R}^k$ over M for an integer $k \geqslant 0$.

Almost complex and stable almost complex manifolds

By an *almost complex* manifold (M^{2n}, J) we mean a smooth manifold M^{2n} with a vector bundle map $J\colon T(M) \to T(M)$ over id_M such that $J^2 = -id_{T(M)}$. In other words, the tangent bundle $T(M)$ admits the structure of a complex vector bundle.

Existence of an almost complex structure J on M is equivalent to existence of a differential 2-form ω on M which is non-degenerate at each point $x \in M$, i.e., for any non-zero vector $v \in T_x(M)$, the 1-form $\{\lambda_x\colon T_x(M) \to \mathbb{R}\}_{x \in M}$ given by $\lambda_x(w) = \omega_x(v, w)$ do not vanish.

By a *stable almost complex* manifold (M, f) we mean a smooth manifold M with a smooth embedding $f\colon M \to E$ into some Euclidean space E such that the normal bundle of the embedding f admits a complex structure or, equivalently, the tangent bundle $T(M)$ to M admits the structure of a complex vector bundle, after adding the product bundle $M \times \mathbb{R}^k$ over M for an integer $k \geqslant 0$.

Orientation of stable almost complex manifolds

Orientation of stable almost complex manifolds

A way of saying that a real vector bundle E over a manifold M is oriented

Orientation of stable almost complex manifolds

A way of saying that a real vector bundle E over a manifold M is oriented is to claim that the structure group of the principal bundle associated to E

Orientation of stable almost complex manifolds

A way of saying that a real vector bundle E over a manifold M is oriented is to claim that the structure group of the principal bundle associated to E is reduced from the group $\mathrm{GL}_n(\mathbb{R})$ to the subgroup $\mathrm{GL}_n^+(\mathbb{R})$ of matrices with positive determinant;

Orientation of stable almost complex manifolds

A way of saying that a real vector bundle E over a manifold M is oriented is to claim that the structure group of the principal bundle associated to E is reduced from the group $\mathrm{GL}_n(\mathbb{R})$ to the subgroup $\mathrm{GL}_n^+(\mathbb{R})$ of matrices with positive determinant; i.e., the classifying map $f\colon M \to B\mathrm{GL}_n(\mathbb{R})$ of E

Orientation of stable almost complex manifolds

A way of saying that a real vector bundle E over a manifold M is oriented is to claim that the structure group of the principal bundle associated to E is reduced from the group $\mathrm{GL}_n(\mathbb{R})$ to the subgroup $\mathrm{GL}_n^+(\mathbb{R})$ of matrices with positive determinant; i.e., the classifying map $f\colon M \to B\mathrm{GL}_n(\mathbb{R})$ of E lifts to a map

$$\widetilde{f}\colon M \to B\mathrm{GL}_n^+(\mathbb{R})$$

Orientation of stable almost complex manifolds

A way of saying that a real vector bundle E over a manifold M is oriented is to claim that the structure group of the principal bundle associated to E is reduced from the group $\mathrm{GL}_n(\mathbb{R})$ to the subgroup $\mathrm{GL}_n^+(\mathbb{R})$ of matrices with positive determinant; i.e., the classifying map $f\colon M \to B\mathrm{GL}_n(\mathbb{R})$ of E lifts to a map

$$\widetilde{f}\colon M \to B\mathrm{GL}_n^+(\mathbb{R})$$

with respect to the map $B\mathrm{GL}_n^+(\mathbb{R}) \to B\mathrm{GL}_n(\mathbb{R})$.

Orientation of stable almost complex manifolds

A way of saying that a real vector bundle E over a manifold M is oriented is to claim that the structure group of the principal bundle associated to E is reduced from the group $\mathrm{GL}_n(\mathbb{R})$ to the subgroup $\mathrm{GL}_n^+(\mathbb{R})$ of matrices with positive determinant; i.e., the classifying map $f\colon M \to B\mathrm{GL}_n(\mathbb{R})$ of E lifts to a map

$$\widetilde{f}\colon M \to B\mathrm{GL}_n^+(\mathbb{R})$$

with respect to the map $B\mathrm{GL}_n^+(\mathbb{R}) \to B\mathrm{GL}_n(\mathbb{R})$.

An orientation on two of the three vector bundles E_1, E_2, and $E_1 \oplus E_2$ over M

Orientation of stable almost complex manifolds

A way of saying that a real vector bundle E over a manifold M is oriented is to claim that the structure group of the principal bundle associated to E is reduced from the group $\mathrm{GL}_n(\mathbb{R})$ to the subgroup $\mathrm{GL}_n^+(\mathbb{R})$ of matrices with positive determinant; i.e., the classifying map $f\colon M \to B\mathrm{GL}_n(\mathbb{R})$ of E lifts to a map

$$\widetilde{f}\colon M \to B\mathrm{GL}_n^+(\mathbb{R})$$

with respect to the map $B\mathrm{GL}_n^+(\mathbb{R}) \to B\mathrm{GL}_n(\mathbb{R})$.

An orientation on two of the three vector bundles E_1, E_2, and $E_1 \oplus E_2$ over M determines an orientation on the third.

Orientation of stable almost complex manifolds

A way of saying that a real vector bundle E over a manifold M is oriented is to claim that the structure group of the principal bundle associated to E is reduced from the group $\mathrm{GL}_n(\mathbb{R})$ to the subgroup $\mathrm{GL}_n^+(\mathbb{R})$ of matrices with positive determinant; i.e., the classifying map $f\colon M \to B\mathrm{GL}_n(\mathbb{R})$ of E lifts to a map

$$\widetilde{f}\colon M \to B\mathrm{GL}_n^+(\mathbb{R})$$

with respect to the map $B\mathrm{GL}_n^+(\mathbb{R}) \to B\mathrm{GL}_n(\mathbb{R})$.

An orientation on two of the three vector bundles E_1, E_2, and $E_1 \oplus E_2$ over M determines an orientation on the third.

By definition, a smooth manifold M is oriented if the tangent bundle $T(M)$ is oriented,

Orientation of stable almost complex manifolds

A way of saying that a real vector bundle E over a manifold M is oriented is to claim that the structure group of the principal bundle associated to E is reduced from the group $\mathrm{GL}_n(\mathbb{R})$ to the subgroup $\mathrm{GL}_n^+(\mathbb{R})$ of matrices with positive determinant; i.e., the classifying map $f\colon M \to B\mathrm{GL}_n(\mathbb{R})$ of E lifts to a map

$$\widetilde{f}\colon M \to B\mathrm{GL}_n^+(\mathbb{R})$$

with respect to the map $B\mathrm{GL}_n^+(\mathbb{R}) \to B\mathrm{GL}_n(\mathbb{R})$.

An orientation on two of the three vector bundles E_1, E_2, and $E_1 \oplus E_2$ over M determines an orientation on the third.

By definition, a smooth manifold M is oriented if the tangent bundle $T(M)$ is oriented, as a real vector bundle over M.

Orientation of stable almost complex manifolds

A way of saying that a real vector bundle E over a manifold M is oriented is to claim that the structure group of the principal bundle associated to E is reduced from the group $\mathrm{GL}_n(\mathbb{R})$ to the subgroup $\mathrm{GL}_n^+(\mathbb{R})$ of matrices with positive determinant; i.e., the classifying map $f\colon M \to B\mathrm{GL}_n(\mathbb{R})$ of E lifts to a map

$$\widetilde{f}\colon M \to B\mathrm{GL}_n^+(\mathbb{R})$$

with respect to the map $B\mathrm{GL}_n^+(\mathbb{R}) \to B\mathrm{GL}_n(\mathbb{R})$.

An orientation on two of the three vector bundles E_1, E_2, and $E_1 \oplus E_2$ over M determines an orientation on the third.

By definition, a smooth manifold M is oriented if the tangent bundle $T(M)$ is oriented, as a real vector bundle over M.

Orientation of stable almost complex manifolds

A way of saying that a real vector bundle E over a manifold M is oriented is to claim that the structure group of the principal bundle associated to E is reduced from the group $\mathrm{GL}_n(\mathbb{R})$ to the subgroup $\mathrm{GL}_n^+(\mathbb{R})$ of matrices with positive determinant; i.e., the classifying map $f\colon M \to B\mathrm{GL}_n(\mathbb{R})$ of E lifts to a map

$$\widetilde{f}\colon M \to B\mathrm{GL}_n^+(\mathbb{R})$$

with respect to the map $B\mathrm{GL}_n^+(\mathbb{R}) \to B\mathrm{GL}_n(\mathbb{R})$.

An orientation on two of the three vector bundles E_1, E_2, and $E_1 \oplus E_2$ over M determines an orientation on the third.

By definition, a smooth manifold M is oriented if the tangent bundle $T(M)$ is oriented, as a real vector bundle over M.

Since any complex vector bundle E over M is oriented,

Orientation of stable almost complex manifolds

A way of saying that a real vector bundle E over a manifold M is oriented is to claim that the structure group of the principal bundle associated to E is reduced from the group $\mathrm{GL}_n(\mathbb{R})$ to the subgroup $\mathrm{GL}_n^+(\mathbb{R})$ of matrices with positive determinant; i.e., the classifying map $f\colon M \to B\mathrm{GL}_n(\mathbb{R})$ of E lifts to a map

$$\widetilde{f}\colon M \to B\mathrm{GL}_n^+(\mathbb{R})$$

with respect to the map $B\mathrm{GL}_n^+(\mathbb{R}) \to B\mathrm{GL}_n(\mathbb{R})$.

An orientation on two of the three vector bundles E_1, E_2, and $E_1 \oplus E_2$ over M determines an orientation on the third.

By definition, a smooth manifold M is oriented if the tangent bundle $T(M)$ is oriented, as a real vector bundle over M.

Since any complex vector bundle E over M is oriented, and the product vector bundle $M \times \mathbb{R}^k$ over M is oriented for any $k \geqslant 1$,

Orientation of stable almost complex manifolds

A way of saying that a real vector bundle E over a manifold M is oriented is to claim that the structure group of the principal bundle associated to E is reduced from the group $\mathrm{GL}_n(\mathbb{R})$ to the subgroup $\mathrm{GL}_n^+(\mathbb{R})$ of matrices with positive determinant; i.e., the classifying map $f\colon M \to B\mathrm{GL}_n(\mathbb{R})$ of E lifts to a map

$$\widetilde{f}\colon M \to B\mathrm{GL}_n^+(\mathbb{R})$$

with respect to the map $B\mathrm{GL}_n^+(\mathbb{R}) \to B\mathrm{GL}_n(\mathbb{R})$.

An orientation on two of the three vector bundles E_1, E_2, and $E_1 \oplus E_2$ over M determines an orientation on the third.

By definition, a smooth manifold M is oriented if the tangent bundle $T(M)$ is oriented, as a real vector bundle over M.

Since any complex vector bundle E over M is oriented, and the product vector bundle $M \times \mathbb{R}^k$ over M is oriented for any $k \geqslant 1$, any stable almost complex manifold M is oriented.

Orientation of stable almost complex manifolds

A way of saying that a real vector bundle E over a manifold M is oriented is to claim that the structure group of the principal bundle associated to E is reduced from the group $\mathrm{GL}_n(\mathbb{R})$ to the subgroup $\mathrm{GL}_n^+(\mathbb{R})$ of matrices with positive determinant; i.e., the classifying map $f\colon M \to B\mathrm{GL}_n(\mathbb{R})$ of E lifts to a map

$$\widetilde{f}\colon M \to B\mathrm{GL}_n^+(\mathbb{R})$$

with respect to the map $B\mathrm{GL}_n^+(\mathbb{R}) \to B\mathrm{GL}_n(\mathbb{R})$.

An orientation on two of the three vector bundles E_1, E_2, and $E_1 \oplus E_2$ over M determines an orientation on the third.

By definition, a smooth manifold M is oriented if the tangent bundle $T(M)$ is oriented, as a real vector bundle over M.

Since any complex vector bundle E over M is oriented, and the product vector bundle $M \times \mathbb{R}^k$ over M is oriented for any $k \geqslant 1$, any stable almost complex manifold M is oriented.

Classifying maps and (stable) almost complex manifolds

An oriented manifold M is almost complex

Classifying maps and (stable) almost complex manifolds

An oriented manifold M is almost complex if and only if the classifying map $f_M \colon M \to B\mathrm{GL}^+_{2n}(\mathbb{R})$ of $T(M)$

Classifying maps and (stable) almost complex manifolds

An oriented manifold M is almost complex if and only if the classifying map $f_M\colon M \to B\mathrm{GL}^+_{2n}(\mathbb{R})$ of $T(M)$ lifts to a map

$$\widetilde{f_M}\colon M \to B\mathrm{GL}_n(\mathbb{C})$$

Classifying maps and (stable) almost complex manifolds

An oriented manifold M is almost complex if and only if the classifying map $f_M \colon M \to B\mathrm{GL}^+_{2n}(\mathbb{R})$ of $T(M)$ lifts to a map

$$\widetilde{f_M} \colon M \to B\mathrm{GL}_n(\mathbb{C})$$

with respect to the forgetful map $B\mathrm{GL}_n(\mathbb{C}) \to B\mathrm{GL}^+_{2n}(\mathbb{R})$.

Classifying maps and (stable) almost complex manifolds

An oriented manifold M is almost complex if and only if the classifying map $f_M\colon M \to B\mathrm{GL}^+_{2n}(\mathbb{R})$ of $T(M)$ lifts to a map

$$\widetilde{f_M}\colon M \to B\mathrm{GL}_n(\mathbb{C})$$

with respect to the forgetful map $B\mathrm{GL}_n(\mathbb{C}) \to B\mathrm{GL}^+_{2n}(\mathbb{R})$.

An oriented manifold M is stable almost complex

Classifying maps and (stable) almost complex manifolds

An oriented manifold M is almost complex if and only if the classifying map $f_M\colon M \to B\mathrm{GL}^+_{2n}(\mathbb{R})$ of $T(M)$ lifts to a map

$$\widetilde{f_M}\colon M \to B\mathrm{GL}_n(\mathbb{C})$$

with respect to the forgetful map $B\mathrm{GL}_n(\mathbb{C}) \to B\mathrm{GL}^+_{2n}(\mathbb{R})$.

An oriented manifold M is stable almost complex if and only if the classifying map $f_M\colon M \to B\mathrm{GL}^+_{2n}(\mathbb{R})$ of $T(M)$

Classifying maps and (stable) almost complex manifolds

An oriented manifold M is almost complex if and only if the classifying map $f_M \colon M \to B\mathrm{GL}^+_{2n}(\mathbb{R})$ of $T(M)$ lifts to a map

$$\widetilde{f_M} \colon M \to B\mathrm{GL}_n(\mathbb{C})$$

with respect to the forgetful map $B\mathrm{GL}_n(\mathbb{C}) \to B\mathrm{GL}^+_{2n}(\mathbb{R})$.

An oriented manifold M is stable almost complex if and only if the classifying map $f_M \colon M \to B\mathrm{GL}^+_{2n}(\mathbb{R})$ of $T(M)$ composed with

$$B\mathrm{GL}^+_{2n}(\mathbb{R}) \to B\mathrm{GL}(\mathbb{R}) = \lim_n B\mathrm{GL}_n(\mathbb{R})$$

Classifying maps and (stable) almost complex manifolds

An oriented manifold M is almost complex if and only if the classifying map $f_M\colon M \to B\mathrm{GL}^+_{2n}(\mathbb{R})$ of $T(M)$ lifts to a map

$$\widetilde{f_M}\colon M \to B\mathrm{GL}_n(\mathbb{C})$$

with respect to the forgetful map $B\mathrm{GL}_n(\mathbb{C}) \to B\mathrm{GL}^+_{2n}(\mathbb{R})$.

An oriented manifold M is stable almost complex if and only if the classifying map $f_M\colon M \to B\mathrm{GL}^+_{2n}(\mathbb{R})$ of $T(M)$ composed with

$$B\mathrm{GL}^+_{2n}(\mathbb{R}) \to B\mathrm{GL}(\mathbb{R}) = \lim_n B\mathrm{GL}_n(\mathbb{R})$$

lifts to a map

$$\widetilde{f_M}\colon M \to B\mathrm{GL}(\mathbb{C}) = \lim_n B\mathrm{GL}_n(\mathbb{C})$$

Classifying maps and (stable) almost complex manifolds

An oriented manifold M is almost complex if and only if the classifying map $f_M \colon M \to BGL^+_{2n}(\mathbb{R})$ of $T(M)$ lifts to a map

$$\widetilde{f_M} \colon M \to BGL_n(\mathbb{C})$$

with respect to the forgetful map $BGL_n(\mathbb{C}) \to BGL^+_{2n}(\mathbb{R})$.

An oriented manifold M is stable almost complex if and only if the classifying map $f_M \colon M \to BGL^+_{2n}(\mathbb{R})$ of $T(M)$ composed with

$$BGL^+_{2n}(\mathbb{R}) \to BGL(\mathbb{R}) = \lim_n BGL_n(\mathbb{R})$$

lifts to a map

$$\widetilde{f_M} \colon M \to BGL(\mathbb{C}) = \lim_n BGL_n(\mathbb{C})$$

with respect to the forgetful map $BGL(\mathbb{C}) \to BGL(\mathbb{R})$.

Classifying maps and (stable) almost complex manifolds

An oriented manifold M is almost complex if and only if the classifying map $f_M\colon M \to B\mathrm{GL}^+_{2n}(\mathbb{R})$ of $T(M)$ lifts to a map

$$\widetilde{f_M}\colon M \to B\mathrm{GL}_n(\mathbb{C})$$

with respect to the forgetful map $B\mathrm{GL}_n(\mathbb{C}) \to B\mathrm{GL}^+_{2n}(\mathbb{R})$.

An oriented manifold M is stable almost complex if and only if the classifying map $f_M\colon M \to B\mathrm{GL}^+_{2n}(\mathbb{R})$ of $T(M)$ composed with

$$B\mathrm{GL}^+_{2n}(\mathbb{R}) \to B\mathrm{GL}(\mathbb{R}) = \lim_n B\mathrm{GL}_n(\mathbb{R})$$

lifts to a map

$$\widetilde{f_M}\colon M \to B\mathrm{GL}(\mathbb{C}) = \lim_n B\mathrm{GL}_n(\mathbb{C})$$

with respect to the forgetful map $B\mathrm{GL}(\mathbb{C}) \to B\mathrm{GL}(\mathbb{R})$.

Symplectic and homotopically symplectic manifolds

By a closed *symplectic* manifold (M^{2n}, ω)

Symplectic and homotopically symplectic manifolds

By a closed *symplectic* manifold (M^{2n}, ω) we mean a closed smooth manifold M^{2n}

Symplectic and homotopically symplectic manifolds

By a closed *symplectic* manifold (M^{2n}, ω) we mean a closed smooth manifold M^{2n} with a smooth 2-form ω

Symplectic and homotopically symplectic manifolds

By a closed *symplectic* manifold (M^{2n}, ω) we mean a closed smooth manifold M^{2n} with a smooth 2-form ω which is both

- closed, i.e., the exterior derivative $d\omega = 0$.

Symplectic and homotopically symplectic manifolds

By a closed *symplectic* manifold (M^{2n}, ω) we mean a closed smooth manifold M^{2n} with a smooth 2-form ω which is both

- closed, i.e., the exterior derivative $d\omega = 0$.
- non-degenerate, i.e., $\omega^n \neq 0$ in $H^{2n}_{\mathrm{dR}}(M^{2n})$.

Symplectic and homotopically symplectic manifolds

By a closed *symplectic* manifold (M^{2n}, ω) we mean a closed smooth manifold M^{2n} with a smooth 2-form ω which is both

- closed, i.e., the exterior derivative $d\omega = 0$.
- non-degenerate, i.e., $\omega^n \neq 0$ in $H^{2n}_{\mathrm{dR}}(M^{2n})$.

Corollary

Any closed symplectic manifold (M^{2n}, ω) admits an almost complex structure.

Symplectic and homotopically symplectic manifolds

By a closed *symplectic* manifold (M^{2n}, ω) we mean a closed smooth manifold M^{2n} with a smooth 2-form ω which is both

- closed, i.e., the exterior derivative $d\omega = 0$.
- non-degenerate, i.e., $\omega^n \neq 0$ in $H^{2n}_{\mathrm{dR}}(M^{2n})$.

Corollary
Any closed symplectic manifold (M^{2n}, ω) admits an almost complex structure. The structure is determined by the 2-form ω.

By a closed *homotopically symplectic* manifold (M^{2n}, ω)

Symplectic and homotopically symplectic manifolds

By a closed *symplectic* manifold (M^{2n}, ω) we mean a closed smooth manifold M^{2n} with a smooth 2-form ω which is both

- closed, i.e., the exterior derivative $d\omega = 0$.
- non-degenerate, i.e., $\omega^n \neq 0$ in $H^{2n}_{\mathrm{dR}}(M^{2n})$.

Corollary

Any closed symplectic manifold (M^{2n}, ω) admits an almost complex structure. The structure is determined by the 2-form ω.

By a closed *homotopically symplectic* manifold (M^{2n}, ω) we mean a closed almost complex manifold M^{2n}

Symplectic and homotopically symplectic manifolds

By a closed *symplectic* manifold (M^{2n}, ω) we mean a closed smooth manifold M^{2n} with a smooth 2-form ω which is both

- closed, i.e., the exterior derivative $d\omega = 0$.
- non-degenerate, i.e., $\omega^n \neq 0$ in $H^{2n}_{\mathrm{dR}}(M^{2n})$.

Corollary

Any closed symplectic manifold (M^{2n}, ω) admits an almost complex structure. The structure is determined by the 2-form ω.

By a closed *homotopically symplectic* manifold (M^{2n}, ω) we mean a closed almost complex manifold M^{2n} with a cohomology class $c \in H^2(M^{2n}; \mathbb{R})$ such that $c^n \neq 0$.

By a closed *symplectic* manifold (M^{2n}, ω) we mean a closed smooth manifold M^{2n} with a smooth 2-form ω which is both

- closed, i.e., the exterior derivative $d\omega = 0$.
- non-degenerate, i.e., $\omega^n \neq 0$ in $H^{2n}_{\mathrm{dR}}(M^{2n})$.

Corollary
Any closed symplectic manifold (M^{2n}, ω) admits an almost complex structure. The structure is determined by the 2-form ω.

By a closed *homotopically symplectic* manifold (M^{2n}, ω) we mean a closed almost complex manifold M^{2n} with a cohomology class $c \in H^2(M^{2n}; \mathbb{R})$ such that $c^n \neq 0$.

Four classes of manifolds

Four classes of manifolds

We have defined the following four classes of manifolds:

Four classes of manifolds

We have defined the following four classes of manifolds:

$$\{\text{symplectic mfds}\} \;\subset\; \{\text{homotopically symplectic mfds}\} \;\subset$$

Four classes of manifolds

We have defined the following four classes of manifolds:

$$\{\text{symplectic mfds}\} \subset \{\text{homotopically symplectic mfds}\} \subset$$

$$\{\text{almost complex mfds}\} \subset \{\text{stable almost complex mfds}\}.$$

Four classes of manifolds

We have defined the following four classes of manifolds:

$$\{\text{symplectic mfds}\} \subset \{\text{homotopically symplectic mfds}\} \subset$$

$$\{\text{almost complex mfds}\} \subset \{\text{stable almost complex mfds}\}.$$

We shall give examples of manifolds showing that all inclusions above are proper.

Four classes of manifolds

We have defined the following four classes of manifolds:

$$\{\text{symplectic mfds}\} \subset \{\text{homotopically symplectic mfds}\} \subset$$
$$\{\text{almost complex mfds}\} \subset \{\text{stable almost complex mfds}\}.$$

We shall give examples of manifolds showing that all inclusions above are proper. Every stable almost complex manifold is also oriented.

Four classes of manifolds

We have defined the following four classes of manifolds:

$$\{\text{symplectic mfds}\} \subset \{\text{homotopically symplectic mfds}\} \subset$$

$$\{\text{almost complex mfds}\} \subset \{\text{stable almost complex mfds}\}.$$

We shall give examples of manifolds showing that all inclusions above are proper. Every stable almost complex manifold is also oriented. We shall see that there are oriented smooth manifolds which are not stable almost complex.

Four classes of manifolds

We have defined the following four classes of manifolds:

$$\{\text{symplectic mfds}\} \subset \{\text{homotopically symplectic mfds}\} \subset$$
$$\{\text{almost complex mfds}\} \subset \{\text{stable almost complex mfds}\}.$$

We shall give examples of manifolds showing that all inclusions above are proper. Every stable almost complex manifold is also oriented. We shall see that there are oriented smooth manifolds which are not stable almost complex.

Oriented non-stable almost complex manifolds

The following result allows us to give examples of smooth oriented connected manifolds which are not stable almost complex.

Oriented non-stable almost complex manifolds

The following result allows us to give examples of smooth oriented connected manifolds which are not stable almost complex.

Math. Nachr. 192 (1998) 159–172.

Oriented non-stable almost complex manifolds

The following result allows us to give examples of smooth oriented connected manifolds which are not stable almost complex.

Math. Nachr. 192 (1998) 159–172.

Corollary (A. Dessai)

Oriented non-stable almost complex manifolds

The following result allows us to give examples of smooth oriented connected manifolds which are not stable almost complex.

Math. Nachr. 192 (1998) 159–172.

Corollary (A. Dessai)
Let M be a closed smooth 10-manifold with $H_1(M;\mathbb{Z}) = 0$

Oriented non-stable almost complex manifolds

The following result allows us to give examples of smooth oriented connected manifolds which are not stable almost complex.

Math. Nachr. 192 (1998) 159–172.

Corollary (A. Dessai)
Let M be a closed smooth 10-manifold with $H_1(M;\mathbb{Z}) = 0$ and such that $H_i(M;\mathbb{Z})$ has no 2-torsion for $i = 1, 2$.

Oriented non-stable almost complex manifolds

The following result allows us to give examples of smooth oriented connected manifolds which are not stable almost complex.

Math. Nachr. 192 (1998) 159–172.

Corollary (A. Dessai)
Let M be a closed smooth 10-manifold with $H_1(M;\mathbb{Z}) = 0$ and such that $H_i(M;\mathbb{Z})$ has no 2-torsion for $i = 1, 2$. Assume also that $H^2(M;\mathbb{Z})$ is generated by an element x such that $x^2 \not\equiv 0 \pmod 2$.

Oriented non-stable almost complex manifolds

The following result allows us to give examples of smooth oriented connected manifolds which are not stable almost complex.

Math. Nachr. 192 (1998) 159–172.

Corollary (A. Dessai)
Let M be a closed smooth 10-manifold with $H_1(M;\mathbb{Z}) = 0$ and such that $H_i(M;\mathbb{Z})$ has no 2-torsion for $i = 1, 2$. Assume also that $H^2(M;\mathbb{Z})$ is generated by an element x such that $x^2 \not\equiv 0 \pmod 2$. Then M admits a stable almost complex structure if and only if

Oriented non-stable almost complex manifolds

The following result allows us to give examples of smooth oriented connected manifolds which are not stable almost complex.

Math. Nachr. 192 (1998) 159–172.

Corollary (A. Dessai)
Let M be a closed smooth 10-manifold with $H_1(M;\mathbb{Z}) = 0$ and such that $H_i(M;\mathbb{Z})$ has no 2-torsion for $i = 1, 2$. Assume also that $H^2(M;\mathbb{Z})$ is generated by an element x such that $x^2 \not\equiv 0 \pmod 2$. Then M admits a stable almost complex structure if and only if $w_2(M) \cdot w_4(M)^2 = 0$.

The following result allows us to give examples of smooth oriented connected manifolds which are not stable almost complex.

Math. Nachr. 192 (1998) 159–172.

Corollary (A. Dessai)
Let M be a closed smooth 10-manifold with $H_1(M;\mathbb{Z}) = 0$ and such that $H_i(M;\mathbb{Z})$ has no 2-torsion for $i = 1, 2$. Assume also that $H^2(M;\mathbb{Z})$ is generated by an element x such that $x^2 \not\equiv 0 \pmod 2$. Then M admits a stable almost complex structure if and only if $w_2(M) \cdot w_4(M)^2 = 0$.

In particular, the existence of a stable almost complex structure on the manifold M above

Oriented non-stable almost complex manifolds

The following result allows us to give examples of smooth oriented connected manifolds which are not stable almost complex.

Math. Nachr. 192 (1998) 159–172.

Corollary (A. Dessai)
Let M be a closed smooth 10-manifold with $H_1(M;\mathbb{Z}) = 0$ and such that $H_i(M;\mathbb{Z})$ has no 2-torsion for $i = 1, 2$. Assume also that $H^2(M;\mathbb{Z})$ is generated by an element x such that $x^2 \not\equiv 0 \pmod 2$. Then M admits a stable almost complex structure if and only if $w_2(M) \cdot w_4(M)^2 = 0$.

In particular, the existence of a stable almost complex structure on the manifold M above depends only on the homotopy type of M.

The following result allows us to give examples of smooth oriented connected manifolds which are not stable almost complex.

Math. Nachr. 192 (1998) 159–172.

Corollary (A. Dessai)
Let M be a closed smooth 10-manifold with $H_1(M;\mathbb{Z}) = 0$ and such that $H_i(M;\mathbb{Z})$ has no 2-torsion for $i = 1, 2$. Assume also that $H^2(M;\mathbb{Z})$ is generated by an element x such that $x^2 \not\equiv 0 \pmod 2$. Then M admits a stable almost complex structure if and only if $w_2(M) \cdot w_4(M)^2 = 0$.

In particular, the existence of a stable almost complex structure on the manifold M above depends only on the homotopy type of M. Once we choose, e.g., a simply connected manifold M

Oriented non-stable almost complex manifolds

The following result allows us to give examples of smooth oriented connected manifolds which are not stable almost complex.

Math. Nachr. 192 (1998) 159–172.

Corollary (A. Dessai)
Let M be a closed smooth 10-manifold with $H_1(M;\mathbb{Z}) = 0$ and such that $H_i(M;\mathbb{Z})$ has no 2-torsion for $i = 1, 2$. Assume also that $H^2(M;\mathbb{Z})$ is generated by an element x such that $x^2 \not\equiv 0 \pmod 2$. Then M admits a stable almost complex structure if and only if $w_2(M) \cdot w_4(M)^2 = 0$.

In particular, the existence of a stable almost complex structure on the manifold M above depends only on the homotopy type of M. Once we choose, e.g., a simply connected manifold M satisfying the hypotheses above,

Oriented non-stable almost complex manifolds

The following result allows us to give examples of smooth oriented connected manifolds which are not stable almost complex.

Math. Nachr. 192 (1998) 159–172.

Corollary (A. Dessai)
Let M be a closed smooth 10-manifold with $H_1(M;\mathbb{Z}) = 0$ and such that $H_i(M;\mathbb{Z})$ has no 2-torsion for $i = 1, 2$. Assume also that $H^2(M;\mathbb{Z})$ is generated by an element x such that $x^2 \not\equiv 0 \pmod 2$. Then M admits a stable almost complex structure if and only if $w_2(M) \cdot w_4(M)^2 = 0$.

In particular, the existence of a stable almost complex structure on the manifold M above depends only on the homotopy type of M. Once we choose, e.g., a simply connected manifold M satisfying the hypotheses above, and such that $w_2(M) \cdot w_4(M)^2 \neq 0$,

Oriented non-stable almost complex manifolds

The following result allows us to give examples of smooth oriented connected manifolds which are not stable almost complex.

Math. Nachr. 192 (1998) 159–172.

Corollary (A. Dessai)
Let M be a closed smooth 10-manifold with $H_1(M;\mathbb{Z}) = 0$ and such that $H_i(M;\mathbb{Z})$ has no 2-torsion for $i = 1, 2$. Assume also that $H^2(M;\mathbb{Z})$ is generated by an element x such that $x^2 \not\equiv 0 \pmod 2$. Then M admits a stable almost complex structure if and only if $w_2(M) \cdot w_4(M)^2 = 0$.

In particular, the existence of a stable almost complex structure on the manifold M above depends only on the homotopy type of M. Once we choose, e.g., a simply connected manifold M satisfying the hypotheses above, and such that $w_2(M) \cdot w_4(M)^2 \neq 0$, we get the required example of a manifold M.

Even dimensional spheres and their products

Even dimensional spheres and their products

Amer. J. Math. 75 (1953) 409–448.

Even dimensional spheres and their products

Amer. J. Math. 75 (1953) 409–448.

Theorem (A. Borel, J.-P. Serre)

Even dimensional spheres and their products

Amer. J. Math. 75 (1953) 409–448.

Theorem (A. Borel, J.-P. Serre)
The spheres S^2 *and* S^6 *are the only spheres*

Even dimensional spheres and their products

Amer. J. Math. 75 (1953) 409–448.

Theorem (A. Borel, J.-P. Serre)
The spheres S^2 and S^6 are the only spheres that admit almost complex structures.

Amer. J. Math. 75 (1953) 409–448.

Theorem (A. Borel, J.-P. Serre)
The spheres S^2 and S^6 are the only spheres that admit almost complex structures.

As $H^2(S^6) = 0$, S^6 is not a homotopically symplectic manifold

Even dimensional spheres and their products

Amer. J. Math. 75 (1953) 409–448.

Theorem (A. Borel, J.-P. Serre)
The spheres S^2 and S^6 are the only spheres that admit almost complex structures.

As $H^2(S^6) = 0$, S^6 is not a homotopically symplectic manifold while $S^2 = \mathbb{C}P^1$ is clearly

Even dimensional spheres and their products

Amer. J. Math. 75 (1953) 409–448.

Theorem (A. Borel, J.-P. Serre)
The spheres S^2 and S^6 are the only spheres that admit almost complex structures.

As $H^2(S^6) = 0$, S^6 is not a homotopically symplectic manifold while $S^2 = \mathbb{C}P^1$ is clearly a symplectic manifold.

Even dimensional spheres and their products

Amer. J. Math. 75 (1953) 409–448.

Theorem (A. Borel, J.-P. Serre)
The spheres S^2 and S^6 are the only spheres that admit almost complex structures.

As $H^2(S^6) = 0$, S^6 is not a homotopically symplectic manifold while $S^2 = \mathbb{C}P^1$ is clearly a symplectic manifold.

Topology Appl. 36 (1990) 39–42.

Even dimensional spheres and their products

Amer. J. Math. 75 (1953) 409–448.

Theorem (A. Borel, J.-P. Serre)
The spheres S^2 and S^6 are the only spheres that admit almost complex structures.

As $H^2(S^6) = 0$, S^6 is not a homotopically symplectic manifold while $S^2 = \mathbb{C}P^1$ is clearly a symplectic manifold.

Topology Appl. 36 (1990) 39–42.

Theorem (B. Datta, S. Subramanian)

Even dimensional spheres and their products

Amer. J. Math. 75 (1953) 409–448.

Theorem (A. Borel, J.-P. Serre)
The spheres S^2 and S^6 are the only spheres that admit almost complex structures.

As $H^2(S^6) = 0$, S^6 is not a homotopically symplectic manifold while $S^2 = \mathbb{C}P^1$ is clearly a symplectic manifold.

Topology Appl. 36 (1990) 39–42.

Theorem (B. Datta, S. Subramanian)
$S^2 \times S^4$, $S^2 \times S^6$, and $S^6 \times S^6$

Even dimensional spheres and their products

Amer. J. Math. 75 (1953) 409–448.

Theorem (A. Borel, J.-P. Serre)
The spheres S^2 and S^6 are the only spheres that admit almost complex structures.

As $H^2(S^6) = 0$, S^6 is not a homotopically symplectic manifold while $S^2 = \mathbb{C}P^1$ is clearly a symplectic manifold.

Topology Appl. 36 (1990) 39–42.

Theorem (B. Datta, S. Subramanian)
$S^2 \times S^4$, $S^2 \times S^6$, and $S^6 \times S^6$ are the only products of even dimensional spheres

Even dimensional spheres and their products

Amer. J. Math. 75 (1953) 409–448.

Theorem (A. Borel, J.-P. Serre)
The spheres S^2 and S^6 are the only spheres that admit almost complex structures.

As $H^2(S^6) = 0$, S^6 is not a homotopically symplectic manifold while $S^2 = \mathbb{C}P^1$ is clearly a symplectic manifold.

Topology Appl. 36 (1990) 39–42.

Theorem (B. Datta, S. Subramanian)
$S^2 \times S^4$, $S^2 \times S^6$, and $S^6 \times S^6$ are the only products of even dimensional spheres that admit almost complex structures.

Even dimensional spheres and their products

Amer. J. Math. 75 (1953) 409–448.

Theorem (A. Borel, J.-P. Serre)
The spheres S^2 and S^6 are the only spheres that admit almost complex structures.

As $H^2(S^6) = 0$, S^6 is not a homotopically symplectic manifold while $S^2 = \mathbb{C}P^1$ is clearly a symplectic manifold.

Topology Appl. 36 (1990) 39–42.

Theorem (B. Datta, S. Subramanian)
$S^2 \times S^4$, $S^2 \times S^6$, and $S^6 \times S^6$ are the only products of even dimensional spheres that admit almost complex structures.

Odd dimensional spheres and their products

By using the principal bundle

$$S^1 \times S^1 \to S^{2m+1} \times S^{2n+1} \to \mathbb{C}P^m \times \mathbb{C}P^n,$$

Odd dimensional spheres and their products

By using the principal bundle

$$S^1 \times S^1 \to S^{2m+1} \times S^{2n+1} \to \mathbb{C}P^m \times \mathbb{C}P^n,$$

we can prove that $S^{2m+1} \times S^{2n+1}$ is a complex manifold

Odd dimensional spheres and their products

By using the principal bundle

$$S^1 \times S^1 \to S^{2m+1} \times S^{2n+1} \to \mathbb{C}P^m \times \mathbb{C}P^n,$$

we can prove that $S^{2m+1} \times S^{2n+1}$ is a complex manifold (H. Hopf, Interscience Publishers, Inc., New York, 1948, pp. 167–185).

Odd dimensional spheres and their products

By using the principal bundle

$$S^1 \times S^1 \to S^{2m+1} \times S^{2n+1} \to \mathbb{C}P^m \times \mathbb{C}P^n,$$

we can prove that $S^{2m+1} \times S^{2n+1}$ is a complex manifold (H. Hopf, Interscience Publishers, Inc., New York, 1948, pp. 167–185).

The Cartesian product $S^{2m+1} \times S^{2n+1}$ of two spheres of odd dimension is a complex manifold

Odd dimensional spheres and their products

By using the principal bundle

$$S^1 \times S^1 \to S^{2m+1} \times S^{2n+1} \to \mathbb{C}P^m \times \mathbb{C}P^n,$$

we can prove that $S^{2m+1} \times S^{2n+1}$ is a complex manifold (H. Hopf, Interscience Publishers, Inc., New York, 1948, pp. 167–185).

The Cartesian product $S^{2m+1} \times S^{2n+1}$ of two spheres of odd dimension is a complex manifold and therefore, the manifold $M = S^{2m+1} \times S^{2n+1}$ is almost complex.

Odd dimensional spheres and their products

By using the principal bundle

$$S^1 \times S^1 \to S^{2m+1} \times S^{2n+1} \to \mathbb{C}P^m \times \mathbb{C}P^n,$$

we can prove that $S^{2m+1} \times S^{2n+1}$ is a complex manifold (H. Hopf, Interscience Publishers, Inc., New York, 1948, pp. 167–185).

The Cartesian product $S^{2m+1} \times S^{2n+1}$ of two spheres of odd dimension is a complex manifold and therefore, the manifold $M = S^{2m+1} \times S^{2n+1}$ is almost complex. Also, if $m, n \geqslant 1$, $H^2(M) = 0$

Odd dimensional spheres and their products

By using the principal bundle

$$S^1 \times S^1 \to S^{2m+1} \times S^{2n+1} \to \mathbb{C}P^m \times \mathbb{C}P^n,$$

we can prove that $S^{2m+1} \times S^{2n+1}$ is a complex manifold (H. Hopf, Interscience Publishers, Inc., New York, 1948, pp. 167–185).

The Cartesian product $S^{2m+1} \times S^{2n+1}$ of two spheres of odd dimension is a complex manifold and therefore, the manifold $M = S^{2m+1} \times S^{2n+1}$ is almost complex. Also, if $m, n \geqslant 1$, $H^2(M) = 0$ and hence, M is not homotopically symplectic.

Odd dimensional spheres and their products

By using the principal bundle

$$S^1 \times S^1 \to S^{2m+1} \times S^{2n+1} \to \mathbb{C}P^m \times \mathbb{C}P^n,$$

we can prove that $S^{2m+1} \times S^{2n+1}$ is a complex manifold (H. Hopf, Interscience Publishers, Inc., New York, 1948, pp. 167–185).

The Cartesian product $S^{2m+1} \times S^{2n+1}$ of two spheres of odd dimension is a complex manifold and therefore, the manifold $M = S^{2m+1} \times S^{2n+1}$ is almost complex. Also, if $m, n \geqslant 1$, $H^2(M) = 0$ and hence, M is not homotopically symplectic.

Odd dimensional spheres are not almost complex by the dimension reason.

Odd dimensional spheres and their products

By using the principal bundle

$$S^1 \times S^1 \to S^{2m+1} \times S^{2n+1} \to \mathbb{C}P^m \times \mathbb{C}P^n,$$

we can prove that $S^{2m+1} \times S^{2n+1}$ is a complex manifold (H. Hopf, Interscience Publishers, Inc., New York, 1948, pp. 167–185).

> The Cartesian product $S^{2m+1} \times S^{2n+1}$ of two spheres of odd dimension is a complex manifold and therefore, the manifold $M = S^{2m+1} \times S^{2n+1}$ is almost complex. Also, if $m, n \geqslant 1$, $H^2(M) = 0$ and hence, M is not homotopically symplectic.

Odd dimensional spheres are not almost complex by the dimension reason. Clearly, all spheres (even and odd dimensional) are stable almost complex because they are stably parallelizable.

Odd dimensional spheres and their products

By using the principal bundle

$$S^1 \times S^1 \to S^{2m+1} \times S^{2n+1} \to \mathbb{C}P^m \times \mathbb{C}P^n,$$

we can prove that $S^{2m+1} \times S^{2n+1}$ is a complex manifold (H. Hopf, Interscience Publishers, Inc., New York, 1948, pp. 167–185).

The Cartesian product $S^{2m+1} \times S^{2n+1}$ of two spheres of odd dimension is a complex manifold and therefore, the manifold $M = S^{2m+1} \times S^{2n+1}$ is almost complex. Also, if $m, n \geqslant 1$, $H^2(M) = 0$ and hence, M is not homotopically symplectic.

Odd dimensional spheres are not almost complex by the dimension reason. Clearly, all spheres (even and odd dimensional) are stable almost complex because they are stably parallelizable.

Examples of stable parallelizable manifolds

- The manifolds S^6 and $S^{2m+1} \times S^{2n+1}$ are almost complex

Examples of stable parallelizable manifolds

- The manifolds S^6 and $S^{2m+1} \times S^{2n+1}$ are almost complex but they are not homotopically symplectic.

Examples of stable parallelizable manifolds

- The manifolds S^6 and $S^{2m+1} \times S^{2n+1}$ are almost complex but they are not homotopically symplectic.

Therefore,

$$\{\text{homotopically symplectic mfds}\} \subset \{\text{almost complex mfds}\}$$

Examples of stable parallelizable manifolds

- The manifolds S^6 and $S^{2m+1} \times S^{2n+1}$ are almost complex but they are not homotopically symplectic.

Therefore,

$$\{\text{homotopically symplectic mfds}\} \subset \{\text{almost complex mfds}\}$$

is a proper inclusion.

- The manifolds S^4 and S^8, S^{10}, ... are stable almost complex

Examples of stable parallelizable manifolds

- The manifolds S^6 and $S^{2m+1} \times S^{2n+1}$ are almost complex but they are not homotopically symplectic.

Therefore,

$$\{\text{homotopically symplectic mfds}\} \subset \{\text{almost complex mfds}\}$$

is a proper inclusion.

- The manifolds S^4 and S^8, S^{10}, ... are stable almost complex but they are not almost complex.

Examples of stable parallelizable manifolds

- The manifolds S^6 and $S^{2m+1} \times S^{2n+1}$ are almost complex but they are not homotopically symplectic.

Therefore,

$$\{\text{homotopically symplectic mfds}\} \subset \{\text{almost complex mfds}\}$$

is a proper inclusion.

- The manifolds S^4 and S^8, S^{10}, ... are stable almost complex but they are not almost complex.

Therefore,

$$\{\text{almost complex mfds}\} \subset \{\text{stable almost complex mfds}\}$$

Examples of stable parallelizable manifolds

- The manifolds S^6 and $S^{2m+1} \times S^{2n+1}$ are almost complex but they are not homotopically symplectic.

Therefore,

$$\{\text{homotopically symplectic mfds}\} \subset \{\text{almost complex mfds}\}$$

is a proper inclusion.

- The manifolds S^4 and $S^8, S^{10}, \ldots$ are stable almost complex but they are not almost complex.

Therefore,

$$\{\text{almost complex mfds}\} \subset \{\text{stable almost complex mfds}\}$$

is a proper inclusion.

Almost complex 4-manifolds

Almost complex 4-manifolds

Actualités Sci. Ind. 1183 (1952)

Almost complex 4-manifolds

Actualités Sci. Ind. 1183 (1952)

Theorem (W.-T. Wu)

Almost complex 4-manifolds

Actualités Sci. Ind. 1183 (1952)

Theorem (W.-T. Wu)
A closed 4-manifolds M is almost complex if and only if

Almost complex 4-manifolds

Actualités Sci. Ind. 1183 (1952)

Theorem (W.-T. Wu)
A closed 4-manifolds M is almost complex if and only if there exists a class $c \in H^2(M, \mathbb{Z})$

Almost complex 4-manifolds

Actualités Sci. Ind. 1183 (1952)

Theorem (W.-T. Wu)
A closed 4-manifolds M is almost complex if and only if there exists a class $c \in H^2(M, \mathbb{Z})$ whose reduction $\mathrm{mod}\ 2$ is $w_2(M)$,

Almost complex 4-manifolds

Actualités Sci. Ind. 1183 (1952)

Theorem (W.-T. Wu)
A closed 4-manifolds M is almost complex if and only if there exists a class $c \in H^2(M, \mathbb{Z})$ whose reduction $\mathrm{mod}\ 2$ is $w_2(M)$, the second Stiefel-Whitney class of M,

Almost complex 4-manifolds

Actualités Sci. Ind. 1183 (1952)

Theorem (W.-T. Wu)
A closed 4-manifolds M is almost complex if and only if there exists a class $c \in H^2(M, \mathbb{Z})$ whose reduction $\mod 2$ is $w_2(M)$, the second Stiefel-Whitney class of M, i.e., $c \equiv w_2(M) \pmod 2$, and

$$c^2 = 2\chi(M) + 3\sigma(M)$$

Almost complex 4-manifolds

Actualités Sci. Ind. 1183 (1952)

Theorem (W.-T. Wu)
A closed 4-manifolds M is almost complex if and only if there exists a class $c \in H^2(M, \mathbb{Z})$ whose reduction $\mathrm{mod}\ 2$ is $w_2(M)$, the second Stiefel-Whitney class of M, i.e., $c \equiv w_2(M) \pmod 2$, and

$$c^2 = 2\chi(M) + 3\sigma(M)$$

for the Euler characteristic $\chi(M)$ and the signature $\sigma(M)$ of M.

Almost complex 4-manifolds

Actualités Sci. Ind. 1183 (1952)

Theorem (W.-T. Wu)
A closed 4-manifolds M is almost complex if and only if there exists a class $c \in H^2(M, \mathbb{Z})$ whose reduction $\mathrm{mod}\ 2$ is $w_2(M)$, the second Stiefel-Whitney class of M, i.e., $c \equiv w_2(M) \pmod 2$, and

$$c^2 = 2\chi(M) + 3\sigma(M)$$

for the Euler characteristic $\chi(M)$ and the signature $\sigma(M)$ of M.

- The connected sum $(S^2 \times S^2)\#(S^2 \times S^2)$ is stable almost complex

Almost complex 4-manifolds

Actualités Sci. Ind. 1183 (1952)

Theorem (W.-T. Wu)
A closed 4-manifolds M is almost complex if and only if there exists a class $c \in H^2(M, \mathbb{Z})$ whose reduction $\mathrm{mod}\ 2$ is $w_2(M)$, the second Stiefel-Whitney class of M, i.e., $c \equiv w_2(M) \pmod 2$, and

$$c^2 = 2\chi(M) + 3\sigma(M)$$

for the Euler characteristic $\chi(M)$ and the signature $\sigma(M)$ of M.

- The connected sum $(S^2 \times S^2)\#(S^2 \times S^2)$ is stable almost complex but it is not almost complex.

Almost complex 4-manifolds

Actualités Sci. Ind. 1183 (1952)

Theorem (W.-T. Wu)
A closed 4-manifolds M is almost complex if and only if there exists a class $c \in H^2(M, \mathbb{Z})$ whose reduction $\mathrm{mod}\ 2$ is $w_2(M)$, the second Stiefel-Whitney class of M, i.e., $c \equiv w_2(M) \pmod 2$, and

$$c^2 = 2\chi(M) + 3\sigma(M)$$

for the Euler characteristic $\chi(M)$ and the signature $\sigma(M)$ of M.

- The connected sum $(S^2 \times S^2)\#(S^2 \times S^2)$ is stable almost complex but it is not almost complex.
- The connected sum $\#_k \mathbb{C}P^2$ is always stable almost complex

Almost complex 4-manifolds

Actualités Sci. Ind. 1183 (1952)

Theorem (W.-T. Wu)
A closed 4-manifolds M is almost complex if and only if there exists a class $c \in H^2(M, \mathbb{Z})$ whose reduction $\bmod 2$ is $w_2(M)$, the second Stiefel-Whitney class of M, i.e., $c \equiv w_2(M) \pmod 2$, and

$$c^2 = 2\chi(M) + 3\sigma(M)$$

for the Euler characteristic $\chi(M)$ and the signature $\sigma(M)$ of M.

- The connected sum $(S^2 \times S^2)\#(S^2 \times S^2)$ is stable almost complex but it is not almost complex.
- The connected sum $\#_k\mathbb{C}P^2$ is always stable almost complex and $\#_k\mathbb{C}P^2$ is almost complex if and only if k is odd. In particular, $\mathbb{C}P^2\#\mathbb{C}P^2$ is not almost complex.

Almost complex connected sum

Almost complex connected sum

Osaka J. Math. 28 (1991) 243–253

Almost complex connected sum

Osaka J. Math. 28 (1991) 243–253

Lemma (Y. Sato)

Almost complex connected sum

Osaka J. Math. 28 (1991) 243–253

Lemma (Y. Sato)
There exists a homology 4-sphere Σ^4 *with* $\pi_1(\Sigma^4) \cong \mathrm{SL}_2(\mathbb{F}_5)$.

Almost complex connected sum

Osaka J. Math. 28 (1991) 243–253

Lemma (Y. Sato)
There exists a homology 4-sphere Σ^4 with $\pi_1(\Sigma^4) \cong \mathrm{SL}_2(\mathbb{F}_5)$.

As we have noted, $\mathbb{C}P^2 \# \mathbb{C}P^2$ is not almost complex.

Almost complex connected sum

Osaka J. Math. 28 (1991) 243–253

Lemma (Y. Sato)
There exists a homology 4-sphere Σ^4 with $\pi_1(\Sigma^4) \cong \mathrm{SL}_2(\mathbb{F}_5)$.

As we have noted, $\mathbb{C}P^2 \# \mathbb{C}P^2$ is not almost complex. However,

- the connected sum $\mathbb{C}P^2 \# \Sigma^4$ is an almost complex manifold

Almost complex connected sum

Osaka J. Math. 28 (1991) 243–253

Lemma (Y. Sato)
There exists a homology 4-sphere Σ^4 *with* $\pi_1(\Sigma^4) \cong \mathrm{SL}_2(\mathbb{F}_5)$.

As we have noted, $\mathbb{C}P^2 \# \mathbb{C}P^2$ is not almost complex. However,

- the connected sum $\mathbb{C}P^2 \# \Sigma^4$ is an almost complex manifold

because the cohomology rings $H^*(\mathbb{C}P^2 \# \Sigma^4)$ and $H^*(\mathbb{C}P^2)$ are isomorphic

Almost complex connected sum

Osaka J. Math. 28 (1991) 243–253

Lemma (Y. Sato)
There exists a homology 4-sphere Σ^4 *with* $\pi_1(\Sigma^4) \cong \mathrm{SL}_2(\mathbb{F}_5)$.

As we have noted, $\mathbb{C}P^2 \# \mathbb{C}P^2$ is not almost complex. However,

- the connected sum $\mathbb{C}P^2 \# \Sigma^4$ is an almost complex manifold

because the cohomology rings $H^*(\mathbb{C}P^2 \# \Sigma^4)$ and $H^*(\mathbb{C}P^2)$ are isomorphic and to apply Wu's theorem to confirm that $\mathbb{C}P^2 \# \Sigma^4$ is almost complex,

Almost complex connected sum

Osaka J. Math. 28 (1991) 243–253

Lemma (Y. Sato)
There exists a homology 4-sphere Σ^4 *with* $\pi_1(\Sigma^4) \cong \mathrm{SL}_2(\mathbb{F}_5)$.

As we have noted, $\mathbb{C}P^2 \# \mathbb{C}P^2$ is not almost complex. However,

- the connected sum $\mathbb{C}P^2 \# \Sigma^4$ is an almost complex manifold

because the cohomology rings $H^*(\mathbb{C}P^2 \# \Sigma^4)$ and $H^*(\mathbb{C}P^2)$ are isomorphic and to apply Wu's theorem to confirm that $\mathbb{C}P^2 \# \Sigma^4$ is almost complex, one may take as $c \in H^*(\mathbb{C}P^2 \# \Sigma^4)$

Almost complex connected sum

Osaka J. Math. 28 (1991) 243–253

Lemma (Y. Sato)
There exists a homology 4-sphere Σ^4 *with* $\pi_1(\Sigma^4) \cong \mathrm{SL}_2(\mathbb{F}_5)$.

As we have noted, $\mathbb{C}P^2 \# \mathbb{C}P^2$ is not almost complex. However,

- the connected sum $\mathbb{C}P^2 \# \Sigma^4$ is an almost complex manifold

because the cohomology rings $H^*(\mathbb{C}P^2 \# \Sigma^4)$ and $H^*(\mathbb{C}P^2)$ are isomorphic and to apply Wu's theorem to confirm that $\mathbb{C}P^2 \# \Sigma^4$ is almost complex, one may take as $c \in H^*(\mathbb{C}P^2 \# \Sigma^4)$ the class corresponding to the first Chern class $c_1(\mathbb{C}P^2)$.

Almost complex connected sum

Osaka J. Math. 28 (1991) 243–253

Lemma (Y. Sato)
There exists a homology 4-sphere Σ^4 *with* $\pi_1(\Sigma^4) \cong \mathrm{SL}_2(\mathbb{F}_5)$.

As we have noted, $\mathbb{C}P^2 \# \mathbb{C}P^2$ is not almost complex. However,

- the connected sum $\mathbb{C}P^2 \# \Sigma^4$ is an almost complex manifold

because the cohomology rings $H^*(\mathbb{C}P^2 \# \Sigma^4)$ and $H^*(\mathbb{C}P^2)$ are isomorphic and to apply Wu's theorem to confirm that $\mathbb{C}P^2 \# \Sigma^4$ is almost complex, one may take as $c \in H^*(\mathbb{C}P^2 \# \Sigma^4)$ the class corresponding to the first Chern class $c_1(\mathbb{C}P^2)$.

Homotopically symplectic connected sum

Homotopically symplectic connected sum

- The connected sum $\mathbb{C}P^2 \# \Sigma^4$ is homotopically symplectic

Homotopically symplectic connected sum

- The connected sum $\mathbb{C}P^2 \# \Sigma^4$ is homotopically symplectic

because it is an almost complex manifold such that

Homotopically symplectic connected sum

- The connected sum $\mathbb{C}P^2 \# \Sigma^4$ is homotopically symplectic

because it is an almost complex manifold such that

$$H^*(\mathbb{C}P^2 \# \Sigma^4; \mathbb{R}) \cong H^*(\mathbb{C}P^2; \mathbb{R})$$

- The connected sum $\mathbb{C}P^2 \# \Sigma^4$ is homotopically symplectic

because it is an almost complex manifold such that

$$H^*(\mathbb{C}P^2 \# \Sigma^4; \mathbb{R}) \cong H^*(\mathbb{C}P^2; \mathbb{R})$$

and there exists an element $c \in H^*(\mathbb{C}P^2 \# \Sigma^4; \mathbb{R})$ with $c^2 \neq 0$,

Homotopically symplectic connected sum

- The connected sum $\mathbb{C}P^2 \# \Sigma^4$ is homotopically symplectic

because it is an almost complex manifold such that

$$H^*(\mathbb{C}P^2 \# \Sigma^4; \mathbb{R}) \cong H^*(\mathbb{C}P^2; \mathbb{R})$$

and there exists an element $c \in H^*(\mathbb{C}P^2 \# \Sigma^4; \mathbb{R})$ with $c^2 \neq 0$, namely the element c corresponding to $c_1(\mathbb{C}P^2)$.

Homotopically symplectic connected sum

- The connected sum $\mathbb{C}P^2 \# \Sigma^4$ is homotopically symplectic

because it is an almost complex manifold such that

$$H^*(\mathbb{C}P^2 \# \Sigma^4; \mathbb{R}) \cong H^*(\mathbb{C}P^2; \mathbb{R})$$

and there exists an element $c \in H^*(\mathbb{C}P^2 \# \Sigma^4; \mathbb{R})$ with $c^2 \neq 0$, namely the element c corresponding to $c_1(\mathbb{C}P^2)$.

We shall see that $\mathbb{C}P^2 \# \Sigma^4$ is not a symplectic manifold.

Homotopically symplectic connected sum

- The connected sum $\mathbb{C}P^2 \# \Sigma^4$ is homotopically symplectic

because it is an almost complex manifold such that

$$H^*(\mathbb{C}P^2 \# \Sigma^4; \mathbb{R}) \cong H^*(\mathbb{C}P^2; \mathbb{R})$$

and there exists an element $c \in H^*(\mathbb{C}P^2 \# \Sigma^4; \mathbb{R})$ with $c^2 \neq 0$, namely the element c corresponding to $c_1(\mathbb{C}P^2)$.

We shall see that $\mathbb{C}P^2 \# \Sigma^4$ is not a symplectic manifold.

Therefore,

$$\{\text{symplectic mfds}\} \subset \{\text{homotopically symplectic mfds}\}$$

Homotopically symplectic connected sum

- The connected sum $\mathbb{C}P^2 \# \Sigma^4$ is homotopically symplectic

because it is an almost complex manifold such that

$$H^*(\mathbb{C}P^2 \# \Sigma^4; \mathbb{R}) \cong H^*(\mathbb{C}P^2; \mathbb{R})$$

and there exists an element $c \in H^*(\mathbb{C}P^2 \# \Sigma^4; \mathbb{R})$ with $c^2 \neq 0$, namely the element c corresponding to $c_1(\mathbb{C}P^2)$.

We shall see that $\mathbb{C}P^2 \# \Sigma^4$ is not a symplectic manifold.

Therefore,

$$\{\text{symplectic mfds}\} \;\subset\; \{\text{homotopically symplectic mfds}\}$$

is a proper inclusion.

Homotopically symplectic connected sum

- The connected sum $\mathbb{C}P^2 \# \Sigma^4$ is homotopically symplectic

because it is an almost complex manifold such that

$$H^*(\mathbb{C}P^2 \# \Sigma^4; \mathbb{R}) \cong H^*(\mathbb{C}P^2; \mathbb{R})$$

and there exists an element $c \in H^*(\mathbb{C}P^2 \# \Sigma^4; \mathbb{R})$ with $c^2 \neq 0$, namely the element c corresponding to $c_1(\mathbb{C}P^2)$.

We shall see that $\mathbb{C}P^2 \# \Sigma^4$ is not a symplectic manifold.

Therefore,

$$\{\text{symplectic mfds}\} \subset \{\text{homotopically symplectic mfds}\}$$

is a proper inclusion.

Taubes Theorem

Taubes Theorem

For a closed 4-manifold X,

Taubes Theorem

For a closed 4-manifold X, let $b_2^+(X)$ be the number of positive entries in a diagonalization of the intersection form of X over $\mathbb{Q}$

Taubes Theorem

For a closed 4-manifold X, let $b_2^+(X)$ be the number of positive entries in a diagonalization of the intersection form of X over $\mathbb{Q}$

$$H^2(X,\mathbb{Q}) \times H^2(X,\mathbb{Q}) \to \mathbb{Q}$$

$$(a,b) \mapsto \langle a \cup b, [X] \rangle.$$

Taubes Theorem

For a closed 4-manifold X, let $b_2^+(X)$ be the number of positive entries in a diagonalization of the intersection form of X over $\mathbb{Q}$

$$H^2(X,\mathbb{Q}) \times H^2(X,\mathbb{Q}) \to \mathbb{Q}$$

$$(a, b) \mapsto \langle a \cup b, [X] \rangle.$$

Math. Research Letters 1 (1994) 809–822

For a closed 4-manifold X, let $b_2^+(X)$ be the number of positive entries in a diagonalization of the intersection form of X over $\mathbb{Q}$

$$H^2(X, \mathbb{Q}) \times H^2(X, \mathbb{Q}) \to \mathbb{Q}$$

$$(a, b) \mapsto \langle a \cup b, [X] \rangle.$$

Math. Research Letters 1 (1994) 809–822

Theorem (M. Taubes)

Taubes Theorem

For a closed 4-manifold X, let $b_2^+(X)$ be the number of positive entries in a diagonalization of the intersection form of X over $\mathbb{Q}$

$$H^2(X, \mathbb{Q}) \times H^2(X, \mathbb{Q}) \to \mathbb{Q}$$

$$(a, b) \mapsto \langle a \cup b, [X] \rangle.$$

Math. Research Letters 1 (1994) 809–822

Theorem (M. Taubes)
Let X and Y be two closed oriented smooth 4-manifolds

Taubes Theorem

For a closed 4-manifold X, let $b_2^+(X)$ be the number of positive entries in a diagonalization of the intersection form of X over $\mathbb{Q}$

$$H^2(X, \mathbb{Q}) \times H^2(X, \mathbb{Q}) \to \mathbb{Q}$$

$$(a, b) \mapsto \langle a \cup b, [X] \rangle.$$

Math. Research Letters 1 (1994) 809–822

Theorem (M. Taubes)
Let X and Y be two closed oriented smooth 4-manifolds such that $b_2^+(X) > 0$ and $b_2^+(Y) > 0$.

Taubes Theorem

For a closed 4-manifold X, let $b_2^+(X)$ be the number of positive entries in a diagonalization of the intersection form of X over $\mathbb{Q}$

$$H^2(X, \mathbb{Q}) \times H^2(X, \mathbb{Q}) \to \mathbb{Q}$$

$$(a, b) \mapsto \langle a \cup b, [X] \rangle.$$

Math. Research Letters 1 (1994) 809–822

Theorem (M. Taubes)
Let X and Y be two closed oriented smooth 4-manifolds such that $b_2^+(X) > 0$ and $b_2^+(Y) > 0$. Then the connected sum $X \# Y$ is not a symplectic manifold.

Taubes Theorem

For a closed 4-manifold X, let $b_2^+(X)$ be the number of positive entries in a diagonalization of the intersection form of X over $\mathbb{Q}$

$$H^2(X,\mathbb{Q}) \times H^2(X,\mathbb{Q}) \to \mathbb{Q}$$

$$(a,b) \mapsto \langle a \cup b, [X] \rangle .$$

Math. Research Letters 1 (1994) 809–822

Theorem (M. Taubes)
Let X and Y be two closed oriented smooth 4-manifolds such that $b_2^+(X) > 0$ and $b_2^+(Y) > 0$. Then the connected sum $X \# Y$ is not a symplectic manifold.

In particular, for $k \geqslant 2$,

Taubes Theorem

For a closed 4-manifold X, let $b_2^+(X)$ be the number of positive entries in a diagonalization of the intersection form of X over $\mathbb{Q}$

$$H^2(X, \mathbb{Q}) \times H^2(X, \mathbb{Q}) \to \mathbb{Q}$$

$$(a, b) \mapsto \langle a \cup b, [X] \rangle.$$

Math. Research Letters 1 (1994) 809–822

Theorem (M. Taubes)
Let X and Y be two closed oriented smooth 4-manifolds such that $b_2^+(X) > 0$ and $b_2^+(Y) > 0$. Then the connected sum $X \# Y$ is not a symplectic manifold.

In particular, for $k \geqslant 2$, the connected sum of k copies of $\mathbb{C}P^2$,

Taubes Theorem

For a closed 4-manifold X, let $b_2^+(X)$ be the number of positive entries in a diagonalization of the intersection form of X over $\mathbb{Q}$

$$H^2(X, \mathbb{Q}) \times H^2(X, \mathbb{Q}) \to \mathbb{Q}$$

$$(a, b) \mapsto \langle a \cup b, [X] \rangle.$$

Math. Research Letters 1 (1994) 809–822

Theorem (M. Taubes)
Let X and Y be two closed oriented smooth 4-manifolds such that $b_2^+(X) > 0$ and $b_2^+(Y) > 0$. Then the connected sum $X \# Y$ is not a symplectic manifold.

In particular, for $k \geqslant 2$, the connected sum of k copies of $\mathbb{C}P^2$,

$$\#_k \mathbb{C}P^2 = \mathbb{C}P^2 \# \cdots \# \mathbb{C}P^2$$

Taubes Theorem

For a closed 4-manifold X, let $b_2^+(X)$ be the number of positive entries in a diagonalization of the intersection form of X over $\mathbb{Q}$

$$H^2(X, \mathbb{Q}) \times H^2(X, \mathbb{Q}) \to \mathbb{Q}$$

$$(a, b) \mapsto \langle a \cup b, [X] \rangle.$$

Math. Research Letters 1 (1994) 809–822

Theorem (M. Taubes)
Let X and Y be two closed oriented smooth 4-manifolds such that $b_2^+(X) > 0$ and $b_2^+(Y) > 0$. Then the connected sum $X \# Y$ is not a symplectic manifold.

In particular, for $k \geqslant 2$, the connected sum of k copies of $\mathbb{C}P^2$,

$$\#_k \mathbb{C}P^2 = \mathbb{C}P^2 \# \cdots \# \mathbb{C}P^2$$

is not a symplectic manifold.

Taubes Theorem

For a closed 4-manifold X, let $b_2^+(X)$ be the number of positive entries in a diagonalization of the intersection form of X over $\mathbb{Q}$

$$H^2(X, \mathbb{Q}) \times H^2(X, \mathbb{Q}) \to \mathbb{Q}$$

$$(a, b) \mapsto \langle a \cup b, [X] \rangle.$$

Math. Research Letters 1 (1994) 809–822

Theorem (M. Taubes)
Let X and Y be two closed oriented smooth 4-manifolds such that $b_2^+(X) > 0$ and $b_2^+(Y) > 0$. Then the connected sum $X \# Y$ is not a symplectic manifold.

In particular, for $k \geqslant 2$, the connected sum of k copies of $\mathbb{C}P^2$,

$$\#_k \mathbb{C}P^2 = \mathbb{C}P^2 \# \cdots \# \mathbb{C}P^2$$

is not a symplectic manifold.

Non-symplectic connected sum

Non-symplectic connected sum

J. Sympl. Geom. Vol. 10 (2012) 17–26.

Non-symplectic connected sum

J. Sympl. Geom. Vol. 10 (2012) 17–26.

Proposition (M. Kaluba, W. Politarczyk)

Non-symplectic connected sum

J. Sympl. Geom. Vol. 10 (2012) 17–26.

Proposition (M. Kaluba, W. Politarczyk)
Let X and M be two closed oriented smooth 4-manifolds

Non-symplectic connected sum

J. Sympl. Geom. Vol. 10 (2012) 17–26.

Proposition (M. Kaluba, W. Politarczyk)
Let X and M be two closed oriented smooth 4-manifolds such that $b_2^+(X) > 0$

Non-symplectic connected sum

J. Sympl. Geom. Vol. 10 (2012) 17–26.

Proposition (M. Kaluba, W. Politarczyk)
Let X and M be two closed oriented smooth 4-manifolds such that $b_2^+(X) > 0$ and $\pi_1(M)$ has a subgroup of finite index $k > 1$.

Non-symplectic connected sum

J. Sympl. Geom. Vol. 10 (2012) 17–26.

Proposition (M. Kaluba, W. Politarczyk)
Let X and M be two closed oriented smooth 4-manifolds such that $b_2^+(X) > 0$ and $\pi_1(M)$ has a subgroup of finite index $k > 1$. Then the connected sum $X \# M$ is not a symplectic manifold.

Non-symplectic connected sum

J. Sympl. Geom. Vol. 10 (2012) 17–26.

Proposition (M. Kaluba, W. Politarczyk)
Let X and M be two closed oriented smooth 4-manifolds such that $b_2^+(X) > 0$ and $\pi_1(M)$ has a subgroup of finite index $k > 1$. Then the connected sum $X \# M$ is not a symplectic manifold.

As $\pi_1(M)$ has a subgroup of finite index $k > 1$,

Non-symplectic connected sum

J. Sympl. Geom. Vol. 10 (2012) 17–26.

Proposition (M. Kaluba, W. Politarczyk)
Let X and M be two closed oriented smooth 4-manifolds such that $b_2^+(X) > 0$ and $\pi_1(M)$ has a subgroup of finite index $k > 1$. Then the connected sum $X \# M$ is not a symplectic manifold.

As $\pi_1(M)$ has a subgroup of finite index $k > 1$, there exists a k-sheeted covering $\widetilde{M} \to M$.

Non-symplectic connected sum

J. Sympl. Geom. Vol. 10 (2012) 17–26.

Proposition (M. Kaluba, W. Politarczyk)
Let X and M be two closed oriented smooth 4-manifolds such that $b_2^+(X) > 0$ and $\pi_1(M)$ has a subgroup of finite index $k > 1$. Then the connected sum $X \# M$ is not a symplectic manifold.

As $\pi_1(M)$ has a subgroup of finite index $k > 1$, there exists a k-sheeted covering $\widetilde{M} \to M$. Then the k-sheeted covering $E \to X \# M$ has the form

Non-symplectic connected sum

J. Sympl. Geom. Vol. 10 (2012) 17–26.

Proposition (M. Kaluba, W. Politarczyk)
Let X and M be two closed oriented smooth 4-manifolds such that $b_2^+(X) > 0$ and $\pi_1(M)$ has a subgroup of finite index $k > 1$. Then the connected sum $X \# M$ is not a symplectic manifold.

As $\pi_1(M)$ has a subgroup of finite index $k > 1$, there exists a k-sheeted covering $\widetilde{M} \to M$. Then the k-sheeted covering $E \to X \# M$ has the form

$$E = kX \# \widetilde{M}$$

Non-symplectic connected sum

J. Sympl. Geom. Vol. 10 (2012) 17–26.

Proposition (M. Kaluba, W. Politarczyk)
Let X and M be two closed oriented smooth 4-manifolds such that $b_2^+(X) > 0$ and $\pi_1(M)$ has a subgroup of finite index $k > 1$. Then the connected sum $X \# M$ is not a symplectic manifold.

As $\pi_1(M)$ has a subgroup of finite index $k > 1$, there exists a k-sheeted covering $\widetilde{M} \to M$. Then the k-sheeted covering $E \to X \# M$ has the form

$$E = kX \# \widetilde{M} = X \# \Big((k-1) X \# \widetilde{M} \Big).$$

Non-symplectic connected sum

J. Sympl. Geom. Vol. 10 (2012) 17–26.

Proposition (M. Kaluba, W. Politarczyk)
Let X and M be two closed oriented smooth 4-manifolds such that $b_2^+(X) > 0$ and $\pi_1(M)$ has a subgroup of finite index $k > 1$. Then the connected sum $X \# M$ is not a symplectic manifold.

As $\pi_1(M)$ has a subgroup of finite index $k > 1$, there exists a k-sheeted covering $\widetilde{M} \to M$. Then the k-sheeted covering $E \to X \# M$ has the form

$$E = kX \# \widetilde{M} = X \# \Big((k-1) X \# \widetilde{M} \Big).$$

Set $Y = (k-1)X \# \widetilde{M}$.

Non-symplectic connected sum

J. Sympl. Geom. Vol. 10 (2012) 17–26.

Proposition (M. Kaluba, W. Politarczyk)
Let X and M be two closed oriented smooth 4-manifolds such that $b_2^+(X) > 0$ and $\pi_1(M)$ has a subgroup of finite index $k > 1$. Then the connected sum $X \# M$ is not a symplectic manifold.

As $\pi_1(M)$ has a subgroup of finite index $k > 1$, there exists a k-sheeted covering $\widetilde{M} \to M$. Then the k-sheeted covering $E \to X \# M$ has the form

$$E = kX \# \widetilde{M} = X \# \Big((k-1)X \# \widetilde{M} \Big).$$

Set $Y = (k-1)X \# \widetilde{M}$. As $b_2^+(Y) \geqslant b_2^+(X) > 0$,

Non-symplectic connected sum

J. Sympl. Geom. Vol. 10 (2012) 17–26.

Proposition (M. Kaluba, W. Politarczyk)
Let X and M be two closed oriented smooth 4-manifolds such that $b_2^+(X) > 0$ and $\pi_1(M)$ has a subgroup of finite index $k > 1$. Then the connected sum $X \# M$ is not a symplectic manifold.

As $\pi_1(M)$ has a subgroup of finite index $k > 1$, there exists a k-sheeted covering $\widetilde{M} \to M$. Then the k-sheeted covering $E \to X \# M$ has the form

$$E = kX \# \widetilde{M} = X \# \Big((k-1)X \# \widetilde{M} \Big).$$

Set $Y = (k-1)X \# \widetilde{M}$. As $b_2^+(Y) \geqslant b_2^+(X) > 0$, it follows by Taubes Theorem (Math. Research Letters 1 (1994) 809–822)

Non-symplectic connected sum

J. Sympl. Geom. Vol. 10 (2012) 17–26.

Proposition (M. Kaluba, W. Politarczyk)
Let X and M be two closed oriented smooth 4-manifolds such that $b_2^+(X) > 0$ and $\pi_1(M)$ has a subgroup of finite index $k > 1$. Then the connected sum $X \# M$ is not a symplectic manifold.

As $\pi_1(M)$ has a subgroup of finite index $k > 1$, there exists a k-sheeted covering $\widetilde{M} \to M$. Then the k-sheeted covering $E \to X \# M$ has the form

$$E = kX \# \widetilde{M} = X \# \Big((k-1)X \# \widetilde{M} \Big).$$

Set $Y = (k-1)X \# \widetilde{M}$. As $b_2^+(Y) \geqslant b_2^+(X) > 0$, it follows by Taubes Theorem (Math. Research Letters 1 (1994) 809–822) that E is not symplectic

Non-symplectic connected sum

J. Sympl. Geom. Vol. 10 (2012) 17–26.

Proposition (M. Kaluba, W. Politarczyk)
Let X and M be two closed oriented smooth 4-manifolds such that $b_2^+(X) > 0$ and $\pi_1(M)$ has a subgroup of finite index $k > 1$. Then the connected sum $X \# M$ is not a symplectic manifold.

As $\pi_1(M)$ has a subgroup of finite index $k > 1$, there exists a k-sheeted covering $\widetilde{M} \to M$. Then the k-sheeted covering $E \to X \# M$ has the form

$$E = kX \# \widetilde{M} = X \# \Big((k-1) X \# \widetilde{M} \Big).$$

Set $Y = (k-1)X \# \widetilde{M}$. As $b_2^+(Y) \geqslant b_2^+(X) > 0$, it follows by Taubes Theorem (Math. Research Letters 1 (1994) 809–822) that E is not symplectic and so, $X \# M$ is not symplectic too.

Kaluba-Politarczyk Theorem

Kaluba-Politarczyk Theorem

J. Sympl. Geom. Vol. 10 (2012) 17–26.

Kaluba-Politarczyk Theorem

J. Sympl. Geom. Vol. 10 (2012) 17–26.

Theorem (M. Kaluba, W. Politarczyk)

Kaluba-Politarczyk Theorem

J. Sympl. Geom. Vol. 10 (2012) 17–26.

Theorem (M. Kaluba, W. Politarczyk)
For any compact Lie group G,

J. Sympl. Geom. Vol. 10 (2012) 17–26.

Theorem (M. Kaluba, W. Politarczyk)
For any compact Lie group G, there exists a smooth action of G on a complex projective space $\mathbb{C}P^n$

Kaluba-Politarczyk Theorem

J. Sympl. Geom. Vol. 10 (2012) 17–26.

Theorem (M. Kaluba, W. Politarczyk)
For any compact Lie group G, there exists a smooth action of G on a complex projective space $\mathbb{C}P^n$ such that the fixed point set contains a connected component

Kaluba-Politarczyk Theorem

J. Sympl. Geom. Vol. 10 (2012) 17–26.

Theorem (M. Kaluba, W. Politarczyk)
For any compact Lie group G, there exists a smooth action of G on a complex projective space $\mathbb{C}P^n$ such that the fixed point set contains a connected component diffeomorphic to $\mathbb{C}P^2 \# \Sigma^4$.

Kaluba-Politarczyk Theorem

J. Sympl. Geom. Vol. 10 (2012) 17–26.

Theorem (M. Kaluba, W. Politarczyk)
For any compact Lie group G, there exists a smooth action of G on a complex projective space $\mathbb{C}P^n$ such that the fixed point set contains a connected component diffeomorphic to $\mathbb{C}P^2 \# \Sigma^4$.

- Construct a smooth action of G on a disk D^{2n+5}

Kaluba-Politarczyk Theorem

J. Sympl. Geom. Vol. 10 (2012) 17–26.

Theorem (M. Kaluba, W. Politarczyk)
For any compact Lie group G, there exists a smooth action of G on a complex projective space $\mathbb{C}P^n$ such that the fixed point set contains a connected component diffeomorphic to $\mathbb{C}P^2 \# \Sigma^4$.

- Construct a smooth action of G on a disk D^{2n+5} with $F(G \circlearrowright D^{2n+5}) \cong \Delta^5$ for a contractible compact smooth manifold Δ^5 of dimension 5,

Kaluba-Politarczyk Theorem

J. Sympl. Geom. Vol. 10 (2012) 17–26.

Theorem (M. Kaluba, W. Politarczyk)
For any compact Lie group G, there exists a smooth action of G on a complex projective space $\mathbb{C}P^n$ such that the fixed point set contains a connected component diffeomorphic to $\mathbb{C}P^2 \# \Sigma^4$.

- Construct a smooth action of G on a disk D^{2n+5} with $F(G \circlearrowright D^{2n+5}) \cong \Delta^5$ for a contractible compact smooth manifold Δ^5 of dimension 5, such that $\partial\Delta^5 = \Sigma^4$.

Kaluba-Politarczyk Theorem

J. Sympl. Geom. Vol. 10 (2012) 17–26.

Theorem (M. Kaluba, W. Politarczyk)
For any compact Lie group G, there exists a smooth action of G on a complex projective space $\mathbb{C}P^n$ such that the fixed point set contains a connected component diffeomorphic to $\mathbb{C}P^2 \# \Sigma^4$.

- Construct a smooth action of G on a disk D^{2n+5} with $F(G \circlearrowright D^{2n+5}) \cong \Delta^5$ for a contractible compact smooth manifold Δ^5 of dimension 5, such that $\partial \Delta^5 = \Sigma^4$.
- The restricted action of G on $\partial D^{2n+5} = S^{2n+4}$

Kaluba-Politarczyk Theorem

J. Sympl. Geom. Vol. 10 (2012) 17–26.

Theorem (M. Kaluba, W. Politarczyk)
For any compact Lie group G, there exists a smooth action of G on a complex projective space $\mathbb{C}P^n$ such that the fixed point set contains a connected component diffeomorphic to $\mathbb{C}P^2 \# \Sigma^4$.

- Construct a smooth action of G on a disk D^{2n+5} with $F(G \circlearrowleft D^{2n+5}) \cong \Delta^5$ for a contractible compact smooth manifold Δ^5 of dimension 5, such that $\partial \Delta^5 = \Sigma^4$.
- The restricted action of G on $\partial D^{2n+5} = S^{2n+4}$ is a smooth action of G on S^{2n+4}

Kaluba-Politarczyk Theorem

J. Sympl. Geom. Vol. 10 (2012) 17–26.

Theorem (M. Kaluba, W. Politarczyk)
For any compact Lie group G, there exists a smooth action of G on a complex projective space $\mathbb{C}P^n$ such that the fixed point set contains a connected component diffeomorphic to $\mathbb{C}P^2 \# \Sigma^4$.

- Construct a smooth action of G on a disk D^{2n+5} with $F(G \circlearrowright D^{2n+5}) \cong \Delta^5$ for a contractible compact smooth manifold Δ^5 of dimension 5, such that $\partial \Delta^5 = \Sigma^4$.
- The restricted action of G on $\partial D^{2n+5} = S^{2n+4}$ is a smooth action of G on S^{2n+4} such that $F(G \circlearrowright S^{2n+4}) \cong \Sigma^4$.

Kaluba-Politarczyk Theorem

J. Sympl. Geom. Vol. 10 (2012) 17–26.

Theorem (M. Kaluba, W. Politarczyk)
For any compact Lie group G, there exists a smooth action of G on a complex projective space $\mathbb{C}P^n$ such that the fixed point set contains a connected component diffeomorphic to $\mathbb{C}P^2 \# \Sigma^4$.

- Construct a smooth action of G on a disk D^{2n+5} with $F(G \circlearrowright D^{2n+5}) \cong \Delta^5$ for a contractible compact smooth manifold Δ^5 of dimension 5, such that $\partial \Delta^5 = \Sigma^4$.
- The restricted action of G on $\partial D^{2n+5} = S^{2n+4}$ is a smooth action of G on S^{2n+4} such that $F(G \circlearrowright S^{2n+4}) \cong \Sigma^4$.
- The G-equivariant connected sum $\mathbb{C}P^{n+2}{}_c\#_x S^{2n+4} \cong \mathbb{C}P^{n+2}$

Kaluba-Politarczyk Theorem

J. Sympl. Geom. Vol. 10 (2012) 17–26.

Theorem (M. Kaluba, W. Politarczyk)
For any compact Lie group G, there exists a smooth action of G on a complex projective space $\mathbb{C}P^n$ such that the fixed point set contains a connected component diffeomorphic to $\mathbb{C}P^2 \# \Sigma^4$.

- Construct a smooth action of G on a disk D^{2n+5} with $F(G \circlearrowright D^{2n+5}) \cong \Delta^5$ for a contractible compact smooth manifold Δ^5 of dimension 5, such that $\partial \Delta^5 = \Sigma^4$.
- The restricted action of G on $\partial D^{2n+5} = S^{2n+4}$ is a smooth action of G on S^{2n+4} such that $F(G \circlearrowright S^{2n+4}) \cong \Sigma^4$.
- The G-equivariant connected sum $\mathbb{C}P^{n+2}{}_c\#_x S^{2n+4} \cong \mathbb{C}P^{n+2}$ at two G-fixed point $c \in \mathbb{C}P^2 \subset \mathbb{C}P^{n+2}$ and $x \in \Sigma^4 \subset S^{2n+4}$

Kaluba-Politarczyk Theorem

J. Sympl. Geom. Vol. 10 (2012) 17–26.

Theorem (M. Kaluba, W. Politarczyk)
For any compact Lie group G, there exists a smooth action of G on a complex projective space $\mathbb{C}P^n$ such that the fixed point set contains a connected component diffeomorphic to $\mathbb{C}P^2 \# \Sigma^4$.

- Construct a smooth action of G on a disk D^{2n+5} with $F(G \circlearrowright D^{2n+5}) \cong \Delta^5$ for a contractible compact smooth manifold Δ^5 of dimension 5, such that $\partial \Delta^5 = \Sigma^4$.
- The restricted action of G on $\partial D^{2n+5} = S^{2n+4}$ is a smooth action of G on S^{2n+4} such that $F(G \circlearrowright S^{2n+4}) \cong \Sigma^4$.
- The G-equivariant connected sum $\mathbb{C}P^{n+2}{}_c\#_x S^{2n+4} \cong \mathbb{C}P^{n+2}$ at two G-fixed point $c \in \mathbb{C}P^2 \subset \mathbb{C}P^{n+2}$ and $x \in \Sigma^4 \subset S^{2n+4}$ yields a smooth action of G on $\mathbb{C}P^{n+2}$

Kaluba-Politarczyk Theorem

J. Sympl. Geom. Vol. 10 (2012) 17–26.

Theorem (M. Kaluba, W. Politarczyk)
For any compact Lie group G, there exists a smooth action of G on a complex projective space $\mathbb{C}P^n$ such that the fixed point set contains a connected component diffeomorphic to $\mathbb{C}P^2 \# \Sigma^4$.

- Construct a smooth action of G on a disk D^{2n+5} with $F(G \circlearrowright D^{2n+5}) \cong \Delta^5$ for a contractible compact smooth manifold Δ^5 of dimension 5, such that $\partial \Delta^5 = \Sigma^4$.
- The restricted action of G on $\partial D^{2n+5} = S^{2n+4}$ is a smooth action of G on S^{2n+4} such that $F(G \circlearrowright S^{2n+4}) \cong \Sigma^4$.
- The G-equivariant connected sum $\mathbb{C}P^{n+2}{}_c\#_x S^{2n+4} \cong \mathbb{C}P^{n+2}$ at two G-fixed point $c \in \mathbb{C}P^2 \subset \mathbb{C}P^{n+2}$ and $x \in \Sigma^4 \subset S^{2n+4}$ yields a smooth action of G on $\mathbb{C}P^{n+2}$ such that

$$F(G \circlearrowright \mathbb{C}P^{n+2}) = \mathbb{C}P^2 \# \Sigma^4.$$

Proof of Kaluba–Politarczyk Theorem

Proof of Kaluba–Politarczyk Theorem

Trans. Amer. Math. Soc. 144 (1969) 67–72

Proof of Kaluba–Politarczyk Theorem

Trans. Amer. Math. Soc. 144 (1969) 67–72

Theorem (M. Kervaire)

Proof of Kaluba–Politarczyk Theorem

Trans. Amer. Math. Soc. 144 (1969) 67–72

Theorem (M. Kervaire)
Any homology 4-sphere is diffeomorphic to the boundary of a compact contractible smooth 5-manifold.

Proof of Kaluba–Politarczyk Theorem

Trans. Amer. Math. Soc. 144 (1969) 67–72

Theorem (M. Kervaire)
Any homology 4-sphere is diffeomorphic to the boundary of a compact contractible smooth 5-manifold.

In particular, Sato's homology 4-sphere Σ^4 bounds a compact contractible smooth 5-manifold Δ^5.

Proof of Kaluba–Politarczyk Theorem

Trans. Amer. Math. Soc. 144 (1969) 67–72

Theorem (M. Kervaire)
Any homology 4-sphere is diffeomorphic to the boundary of a compact contractible smooth 5-manifold.

In particular, Sato's homology 4-sphere Σ^4 bounds a compact contractible smooth 5-manifold Δ^5. As $\pi_1(\partial\Delta^5) \cong \mathrm{SL}_2(\mathbb{F}_5)$,

Proof of Kaluba–Politarczyk Theorem

Trans. Amer. Math. Soc. 144 (1969) 67–72

Theorem (M. Kervaire)
Any homology 4-sphere is diffeomorphic to the boundary of a compact contractible smooth 5-manifold.

In particular, Sato's homology 4-sphere Σ^4 bounds a compact contractible smooth 5-manifold Δ^5. As $\pi_1(\partial\Delta^5) \cong \mathrm{SL}_2(\mathbb{F}_5)$, Δ^5 is not homeomorphic to D^5.

Proof of Kaluba–Politarczyk Theorem

Trans. Amer. Math. Soc. 144 (1969) 67–72

Theorem (M. Kervaire)
Any homology 4-sphere is diffeomorphic to the boundary of a compact contractible smooth 5-manifold.

In particular, Sato's homology 4-sphere Σ^4 bounds a compact contractible smooth 5-manifold Δ^5. As $\pi_1(\partial\Delta^5) \cong \mathrm{SL}_2(\mathbb{F}_5)$, Δ^5 is not homeomorphic to D^5.

Let G be a compact Lie group.

Proof of Kaluba–Politarczyk Theorem

Trans. Amer. Math. Soc. 144 (1969) 67–72

Theorem (M. Kervaire)
Any homology 4-sphere is diffeomorphic to the boundary of a compact contractible smooth 5-manifold.

In particular, Sato's homology 4-sphere Σ^4 bounds a compact contractible smooth 5-manifold Δ^5. As $\pi_1(\partial\Delta^5) \cong \mathrm{SL}_2(\mathbb{F}_5)$, Δ^5 is not homeomorphic to D^5.

Let G be a compact Lie group. As Δ^5 is contractible,

Proof of Kaluba–Politarczyk Theorem

Trans. Amer. Math. Soc. 144 (1969) 67–72

Theorem (M. Kervaire)
Any homology 4-sphere is diffeomorphic to the boundary of a compact contractible smooth 5-manifold.

In particular, Sato's homology 4-sphere Σ^4 bounds a compact contractible smooth 5-manifold Δ^5. As $\pi_1(\partial\Delta^5) \cong \mathrm{SL}_2(\mathbb{F}_5)$, Δ^5 is not homeomorphic to D^5.

Let G be a compact Lie group. As Δ^5 is contractible, so is the join $X = G * \Delta^5$,

Proof of Kaluba–Politarczyk Theorem

Trans. Amer. Math. Soc. 144 (1969) 67–72

Theorem (M. Kervaire)
Any homology 4-sphere is diffeomorphic to the boundary of a compact contractible smooth 5-manifold.

In particular, Sato's homology 4-sphere Σ^4 bounds a compact contractible smooth 5-manifold Δ^5. As $\pi_1(\partial\Delta^5) \cong \mathrm{SL}_2(\mathbb{F}_5)$, Δ^5 is not homeomorphic to D^5.

Let G be a compact Lie group. As Δ^5 is contractible, so is the join $X = G * \Delta^5$, which admits the structure of a finite G-CW complex such that $F(G \circlearrowright X) = \Delta^5$.

Proof of Kaluba–Politarczyk Theorem

Trans. Amer. Math. Soc. 144 (1969) 67–72

Theorem (M. Kervaire)
Any homology 4-sphere is diffeomorphic to the boundary of a compact contractible smooth 5-manifold.

In particular, Sato's homology 4-sphere Σ^4 bounds a compact contractible smooth 5-manifold Δ^5. As $\pi_1(\partial\Delta^5) \cong \mathrm{SL}_2(\mathbb{F}_5)$, Δ^5 is not homeomorphic to D^5.

Let G be a compact Lie group. As Δ^5 is contractible, so is the join $X = G * \Delta^5$, which admits the structure of a finite G-CW complex such that $F(G \circlearrowright X) = \Delta^5$. By using the product G-vector bundle $X \times V^n$ over X

Proof of Kaluba–Politarczyk Theorem

Trans. Amer. Math. Soc. 144 (1969) 67–72

Theorem (M. Kervaire)
Any homology 4-sphere is diffeomorphic to the boundary of a compact contractible smooth 5-manifold.

In particular, Sato's homology 4-sphere Σ^4 bounds a compact contractible smooth 5-manifold Δ^5. As $\pi_1(\partial\Delta^5) \cong \mathrm{SL}_2(\mathbb{F}_5)$, Δ^5 is not homeomorphic to D^5.

Let G be a compact Lie group. As Δ^5 is contractible, so is the join $X = G * \Delta^5$, which admits the structure of a finite G-CW complex such that $F(G \circlearrowright X) = \Delta^5$. By using the product G-vector bundle $X \times V^n$ over X for an appropriate complex G-module V^n,

Proof of Kaluba–Politarczyk Theorem

Trans. Amer. Math. Soc. 144 (1969) 67–72

Theorem (M. Kervaire)
Any homology 4*-sphere is diffeomorphic to the boundary of a compact contractible smooth* 5*-manifold.*

In particular, Sato's homology 4-sphere Σ^4 bounds a compact contractible smooth 5-manifold Δ^5. As $\pi_1(\partial\Delta^5) \cong \mathrm{SL}_2(\mathbb{F}_5)$, Δ^5 is not homeomorphic to D^5.

Let G be a compact Lie group. As Δ^5 is contractible, so is the join $X = G * \Delta^5$, which admits the structure of a finite G-CW complex such that $F(G \circlearrowright X) = \Delta^5$. By using the product G-vector bundle $X \times V^n$ over X for an appropriate complex G-module V^n, we can thicken up X into a disk D^{2n+5}

Proof of Kaluba–Politarczyk Theorem

Trans. Amer. Math. Soc. 144 (1969) 67–72

Theorem (M. Kervaire)
Any homology 4*-sphere is diffeomorphic to the boundary of a compact contractible smooth* 5*-manifold.*

In particular, Sato's homology 4-sphere Σ^4 bounds a compact contractible smooth 5-manifold Δ^5. As $\pi_1(\partial\Delta^5) \cong \mathrm{SL}_2(\mathbb{F}_5)$, Δ^5 is not homeomorphic to D^5.

Let G be a compact Lie group. As Δ^5 is contractible, so is the join $X = G * \Delta^5$, which admits the structure of a finite G-CW complex such that $F(G \circlearrowright X) = \Delta^5$. By using the product G-vector bundle $X \times V^n$ over X for an appropriate complex G-module V^n, we can thicken up X into a disk D^{2n+5} equipped with a smooth action of G

Proof of Kaluba–Politarczyk Theorem

Trans. Amer. Math. Soc. 144 (1969) 67–72

Theorem (M. Kervaire)
Any homology 4-sphere is diffeomorphic to the boundary of a compact contractible smooth 5-manifold.

In particular, Sato's homology 4-sphere Σ^4 bounds a compact contractible smooth 5-manifold Δ^5. As $\pi_1(\partial\Delta^5) \cong \mathrm{SL}_2(\mathbb{F}_5)$, Δ^5 is not homeomorphic to D^5.

Let G be a compact Lie group. As Δ^5 is contractible, so is the join $X = G * \Delta^5$, which admits the structure of a finite G-CW complex such that $F(G \circlearrowright X) = \Delta^5$. By using the product G-vector bundle $X \times V^n$ over X for an appropriate complex G-module V^n, we can thicken up X into a disk D^{2n+5} equipped with a smooth action of G such that $F(G \circlearrowright D^{2n+5}) = \Delta^5$

Proof of Kaluba–Politarczyk Theorem

Trans. Amer. Math. Soc. 144 (1969) 67–72

Theorem (M. Kervaire)
Any homology 4-sphere is diffeomorphic to the boundary of a compact contractible smooth 5-manifold.

In particular, Sato's homology 4-sphere Σ^4 bounds a compact contractible smooth 5-manifold Δ^5. As $\pi_1(\partial\Delta^5) \cong \mathrm{SL}_2(\mathbb{F}_5)$, Δ^5 is not homeomorphic to D^5.

Let G be a compact Lie group. As Δ^5 is contractible, so is the join $X = G * \Delta^5$, which admits the structure of a finite G-CW complex such that $F(G \circlearrowright X) = \Delta^5$. By using the product G-vector bundle $X \times V^n$ over X for an appropriate complex G-module V^n, we can thicken up X into a disk D^{2n+5} equipped with a smooth action of G such that $F(G \circlearrowright D^{2n+5}) = \Delta^5$ and at any $x \in F(G \circlearrowright D^{2n+5})$,

Proof of Kaluba–Politarczyk Theorem

Trans. Amer. Math. Soc. 144 (1969) 67–72

Theorem (M. Kervaire)
Any homology 4*-sphere is diffeomorphic to the boundary of a compact contractible smooth* 5*-manifold.*

In particular, Sato's homology 4-sphere Σ^4 bounds a compact contractible smooth 5-manifold Δ^5. As $\pi_1(\partial\Delta^5) \cong \mathrm{SL}_2(\mathbb{F}_5)$, Δ^5 is not homeomorphic to D^5.

Let G be a compact Lie group. As Δ^5 is contractible, so is the join $X = G * \Delta^5$, which admits the structure of a finite G-CW complex such that $F(G \circlearrowright X) = \Delta^5$. By using the product G-vector bundle $X \times V^n$ over X for an appropriate complex G-module V^n, we can thicken up X into a disk D^{2n+5} equipped with a smooth action of G such that $F(G \circlearrowright D^{2n+5}) = \Delta^5$ and at any $x \in F(G \circlearrowright D^{2n+5})$, the normal G-module is the realification of V^n.

Proof of Kaluba–Politarczyk Theorem

Therefore, we obtain a smooth action of G on the sphere S^{2n+4}

Proof of Kaluba–Politarczyk Theorem

Therefore, we obtain a smooth action of G on the sphere S^{2n+4} such that $F(G \circlearrowright S^{2n+4})$ is diffeomorphic to Σ^4

Proof of Kaluba–Politarczyk Theorem

Therefore, we obtain a smooth action of G on the sphere S^{2n+4} such that $F(G \circlearrowright S^{2n+4})$ is diffeomorphic to Σ^4 and at any point $x \in F(G \circlearrowright S^{2n+4})$,

Therefore, we obtain a smooth action of G on the sphere S^{2n+4} such that $F(G \circlearrowright S^{2n+4})$ is diffeomorphic to Σ^4 and at any point $x \in F(G \circlearrowright S^{2n+4})$, the normal G-module is the realification of V^n.

Proof of Kaluba–Politarczyk Theorem

Therefore, we obtain a smooth action of G on the sphere S^{2n+4} such that $F(G \circlearrowright S^{2n+4})$ is diffeomorphic to Σ^4 and at any point $x \in F(G \circlearrowright S^{2n+4})$, the normal G-module is the realification of V^n.

Therefore, we obtain a smooth action of G on the sphere S^{2n+4} such that $F(G \circlearrowright S^{2n+4})$ is diffeomorphic to Σ^4 and at any point $x \in F(G \circlearrowright S^{2n+4})$, the normal G-module is the realification of V^n.

Let $S^{2n+5} = S(V^n \oplus \mathbb{C}^3)$ be equipped with the linear action of G,

Proof of Kaluba–Politarczyk Theorem

Therefore, we obtain a smooth action of G on the sphere S^{2n+4} such that $F(G \circlearrowright S^{2n+4})$ is diffeomorphic to Σ^4 and at any point $x \in F(G \circlearrowright S^{2n+4})$, the normal G-module is the realification of V^n.

Let $S^{2n+5} = S(V^n \oplus \mathbb{C}^3)$ be equipped with the linear action of G, where G acts trivially on $\mathbb{C}^3$.

Proof of Kaluba–Politarczyk Theorem

Therefore, we obtain a smooth action of G on the sphere S^{2n+4} such that $F(G \circlearrowright S^{2n+4})$ is diffeomorphic to Σ^4 and at any point $x \in F(G \circlearrowright S^{2n+4})$, the normal G-module is the realification of V^n.

Let $S^{2n+5} = S(V^n \oplus \mathbb{C}^3)$ be equipped with the linear action of G, where G acts trivially on $\mathbb{C}^3$. Then $\mathbb{C}P^{n+2} = S(V^n \oplus \mathbb{C}^3)/S^1$

Proof of Kaluba–Politarczyk Theorem

Therefore, we obtain a smooth action of G on the sphere S^{2n+4} such that $F(G \circlearrowright S^{2n+4})$ is diffeomorphic to Σ^4 and at any point $x \in F(G \circlearrowright S^{2n+4})$, the normal G-module is the realification of V^n.

Let $S^{2n+5} = S(V^n \oplus \mathbb{C}^3)$ be equipped with the linear action of G, where G acts trivially on $\mathbb{C}^3$. Then $\mathbb{C}P^{n+2} = S(V^n \oplus \mathbb{C}^3)/S^1$ has a smooth action of G

Proof of Kaluba–Politarczyk Theorem

Therefore, we obtain a smooth action of G on the sphere S^{2n+4} such that $F(G \circlearrowright S^{2n+4})$ is diffeomorphic to Σ^4 and at any point $x \in F(G \circlearrowright S^{2n+4})$, the normal G-module is the realification of V^n.

Let $S^{2n+5} = S(V^n \oplus \mathbb{C}^3)$ be equipped with the linear action of G, where G acts trivially on $\mathbb{C}^3$. Then $\mathbb{C}P^{n+2} = S(V^n \oplus \mathbb{C}^3)/S^1$ has a smooth action of G such that $F(G \circlearrowright \mathbb{C}P^{n+2}) \supset \mathbb{C}P^2$ as some connected component

Proof of Kaluba–Politarczyk Theorem

Therefore, we obtain a smooth action of G on the sphere S^{2n+4} such that $F(G \circlearrowright S^{2n+4})$ is diffeomorphic to Σ^4 and at any point $x \in F(G \circlearrowright S^{2n+4})$, the normal G-module is the realification of V^n.

Let $S^{2n+5} = S(V^n \oplus \mathbb{C}^3)$ be equipped with the linear action of G, where G acts trivially on $\mathbb{C}^3$. Then $\mathbb{C}P^{n+2} = S(V^n \oplus \mathbb{C}^3)/S^1$ has a smooth action of G such that $F(G \circlearrowright \mathbb{C}P^{n+2}) \supset \mathbb{C}P^2$ as some connected component and at any point $c \in \mathbb{C}P^2$,

Proof of Kaluba–Politarczyk Theorem

Therefore, we obtain a smooth action of G on the sphere S^{2n+4} such that $F(G \circlearrowright S^{2n+4})$ is diffeomorphic to Σ^4 and at any point $x \in F(G \circlearrowright S^{2n+4})$, the normal G-module is the realification of V^n.

Let $S^{2n+5} = S(V^n \oplus \mathbb{C}^3)$ be equipped with the linear action of G, where G acts trivially on $\mathbb{C}^3$. Then $\mathbb{C}P^{n+2} = S(V^n \oplus \mathbb{C}^3)/S^1$ has a smooth action of G such that $F(G \circlearrowright \mathbb{C}P^{n+2}) \supset \mathbb{C}P^2$ as some connected component and at any point $c \in \mathbb{C}P^2$, the normal G-module is the realification of V^n.

Proof of Kaluba–Politarczyk Theorem

Therefore, we obtain a smooth action of G on the sphere S^{2n+4} such that $F(G \circlearrowright S^{2n+4})$ is diffeomorphic to Σ^4 and at any point $x \in F(G \circlearrowright S^{2n+4})$, the normal G-module is the realification of V^n.

Let $S^{2n+5} = S(V^n \oplus \mathbb{C}^3)$ be equipped with the linear action of G, where G acts trivially on $\mathbb{C}^3$. Then $\mathbb{C}P^{n+2} = S(V^n \oplus \mathbb{C}^3)/S^1$ has a smooth action of G such that $F(G \circlearrowright \mathbb{C}P^{n+2}) \supset \mathbb{C}P^2$ as some connected component and at any point $c \in \mathbb{C}P^2$, the normal G-module is the realification of V^n.

By forming the G-equivariant connected sum at x and c,

$$\mathbb{C}P^{n+2}{}_c\#_x S^{2n+4} \cong \mathbb{C}P^{n+2}$$

Proof of Kaluba–Politarczyk Theorem

Therefore, we obtain a smooth action of G on the sphere S^{2n+4} such that $F(G \circlearrowright S^{2n+4})$ is diffeomorphic to Σ^4 and at any point $x \in F(G \circlearrowright S^{2n+4})$, the normal G-module is the realification of V^n.

Let $S^{2n+5} = S(V^n \oplus \mathbb{C}^3)$ be equipped with the linear action of G, where G acts trivially on $\mathbb{C}^3$. Then $\mathbb{C}P^{n+2} = S(V^n \oplus \mathbb{C}^3)/S^1$ has a smooth action of G such that $F(G \circlearrowright \mathbb{C}P^{n+2}) \supset \mathbb{C}P^2$ as some connected component and at any point $c \in \mathbb{C}P^2$, the normal G-module is the realification of V^n.

By forming the G-equivariant connected sum at x and c,

$$\mathbb{C}P^{n+2}{}_c\#_x S^{2n+4} \cong \mathbb{C}P^{n+2}$$

we get a smooth action of G on $\mathbb{C}P^{n+2}$ such that $F(G \circlearrowright \mathbb{C}P^{n+2})$

Proof of Kaluba–Politarczyk Theorem

Therefore, we obtain a smooth action of G on the sphere S^{2n+4} such that $F(G \circlearrowright S^{2n+4})$ is diffeomorphic to Σ^4 and at any point $x \in F(G \circlearrowright S^{2n+4})$, the normal G-module is the realification of V^n.

Let $S^{2n+5} = S(V^n \oplus \mathbb{C}^3)$ be equipped with the linear action of G, where G acts trivially on $\mathbb{C}^3$. Then $\mathbb{C}P^{n+2} = S(V^n \oplus \mathbb{C}^3)/S^1$ has a smooth action of G such that $F(G \circlearrowright \mathbb{C}P^{n+2}) \supset \mathbb{C}P^2$ as some connected component and at any point $c \in \mathbb{C}P^2$, the normal G-module is the realification of V^n.

By forming the G-equivariant connected sum at x and c,

$$\mathbb{C}P^{n+2}{}_c\#_x S^{2n+4} \cong \mathbb{C}P^{n+2}$$

we get a smooth action of G on $\mathbb{C}P^{n+2}$ such that $F(G \circlearrowright \mathbb{C}P^{n+2})$ contains a connected component diffeomorphic to $\mathbb{C}P^2 \# \Sigma^4$. □

Main Theorem

Main Theorem

Main Theorem

Let G be a compact Lie group such that G_0 is nonabelian

Main Theorem

Let G be a compact Lie group such that G_0 is nonabelian or G/G_0 is not of prime power order.

Main Theorem

Let G be a compact Lie group such that G_0 is nonabelian or G/G_0 is not of prime power order. There exists a smooth action of G on some complex projective space $\mathbb{C}P^{d+n}$

Main Theorem

Let G be a compact Lie group such that G_0 is nonabelian or G/G_0 is not of prime power order. There exists a smooth action of G on some complex projective space $\mathbb{C}P^{d+n}$ such that the fixed point set $F(G \circlearrowright \mathbb{C}P^{d+n})$

Main Theorem

Let G be a compact Lie group such that G_0 is nonabelian or G/G_0 is not of prime power order. There exists a smooth action of G on some complex projective space $\mathbb{C}P^{d+n}$ such that the fixed point set $F(G \circlearrowright \mathbb{C}P^{d+n})$ contains connected components M of dimension $2d$ which are:

Main Theorem

Main Theorem

Let G be a compact Lie group such that G_0 is nonabelian or G/G_0 is not of prime power order. There exists a smooth action of G on some complex projective space $\mathbb{C}P^{d+n}$ such that the fixed point set $F(G \circlearrowright \mathbb{C}P^{d+n})$ contains connected components M of dimension $2d$ which are:

- *stable almost complex and not almost complex,*

Main Theorem

Main Theorem

Let G be a compact Lie group such that G_0 is nonabelian or G/G_0 is not of prime power order. There exists a smooth action of G on some complex projective space $\mathbb{C}P^{d+n}$ such that the fixed point set $F(G \circlearrowright \mathbb{C}P^{d+n})$ contains connected components M of dimension $2d$ which are:

- *stable almost complex and not almost complex, e.g., S^4, S^8, S^{10}, S^{12}, ...,*

Main Theorem

Let G be a compact Lie group such that G_0 is nonabelian or G/G_0 is not of prime power order. There exists a smooth action of G on some complex projective space $\mathbb{C}P^{d+n}$ such that the fixed point set $F(G \circlearrowright \mathbb{C}P^{d+n})$ contains connected components M of dimension $2d$ which are:

- *stable almost complex* and not *almost complex*, e.g., S^4, S^8, S^{10}, S^{12}, $\ldots$,
- *almost complex* and not *homotopically symplectic*,

Main Theorem

Let G be a compact Lie group such that G_0 is nonabelian or G/G_0 is not of prime power order. There exists a smooth action of G on some complex projective space $\mathbb{C}P^{d+n}$ such that the fixed point set $F(G \circlearrowright \mathbb{C}P^{d+n})$ contains connected components M of dimension $2d$ which are:

- *stable almost complex and not almost complex,* *e.g., S^4, S^8, S^{10}, S^{12}, $\ldots$,*
- *almost complex and not homotopically symplectic,* *e.g., S^6, $S^2 \times S^4$, $S^2 \times S^6$, $S^6 \times S^6$, and $S^{2m+1} \times S^{2n+1}$,*
- *homotopically symplectic and not symplectic,*

Main Theorem

Main Theorem
Let G be a compact Lie group such that G_0 is nonabelian or G/G_0 is not of prime power order. There exists a smooth action of G on some complex projective space $\mathbb{C}P^{d+n}$ such that the fixed point set $F(G \circlearrowright \mathbb{C}P^{d+n})$ contains connected components M of dimension $2d$ which are:

- *stable almost complex and not almost complex, e.g., S^4, S^8, S^{10}, S^{12}, $\ldots$,*
- *almost complex and not homotopically symplectic, e.g., S^6, $S^2 \times S^4$, $S^2 \times S^6$, $S^6 \times S^6$, and $S^{2m+1} \times S^{2n+1}$,*
- *homotopically symplectic and not symplectic, e.g., $\mathbb{C}P^2 \# \Sigma^4$, where Σ^4 is Sato's homology sphere.*

Main Theorem

Main Theorem

Let G be a compact Lie group such that G_0 is nonabelian or G/G_0 is not of prime power order. There exists a smooth action of G on some complex projective space $\mathbb{C}P^{d+n}$ such that the fixed point set $F(G \circlearrowright \mathbb{C}P^{d+n})$ contains connected components M of dimension $2d$ which are:

- *stable almost complex and not almost complex, e.g., S^4, S^8, S^{10}, S^{12}, $\ldots$,*
- *almost complex and not homotopically symplectic, e.g., S^6, $S^2 \times S^4$, $S^2 \times S^6$, $S^6 \times S^6$, and $S^{2m+1} \times S^{2n+1}$,*
- *homotopically symplectic and not symplectic, e.g., $\mathbb{C}P^2 \# \Sigma^4$, where Σ^4 is Sato's homology sphere.*

The idea of the proof of Main Theorem

The idea of the proof of Main Theorem

The case where a connected component M of the fixed point set

The idea of the proof of Main Theorem

The case where a connected component M of the fixed point set $F(G \circlearrowright \mathbb{C}P^{d+n})$ is homotopically symplectic and not symplectic

The idea of the proof of Main Theorem

The case where a connected component M of the fixed point set $F(G \circlearrowright \mathbb{C}P^{d+n})$ is homotopically symplectic and not symplectic is covered by the Kaluba-Politarczyk Theorem.

The idea of the proof of Main Theorem

The case where a connected component M of the fixed point set $F(G \circlearrowright \mathbb{C}P^{d+n})$ is homotopically symplectic and not symplectic is covered by the Kaluba-Politarczyk Theorem. In the remaining two cases of M,

The idea of the proof of Main Theorem

The case where a connected component M of the fixed point set $F(G \circlearrowright \mathbb{C}P^{d+n})$ is homotopically symplectic and not symplectic is covered by the Kaluba-Politarczyk Theorem. In the remaining two cases of M, we argue as follows.

The idea of the proof of Main Theorem

The case where a connected component M of the fixed point set $F(G \circlearrowright \mathbb{C}P^{d+n})$ is homotopically symplectic and not symplectic is covered by the Kaluba-Politarczyk Theorem. In the remaining two cases of M, we argue as follows.

- First, we construct a smooth action of G on S^{2d+2n} such that

The idea of the proof of Main Theorem

The case where a connected component M of the fixed point set $F(G \circlearrowright \mathbb{C}P^{d+n})$ is homotopically symplectic and not symplectic is covered by the Kaluba-Politarczyk Theorem. In the remaining two cases of M, we argue as follows.

- First, we construct a smooth action of G on S^{2d+2n} such that $F(G \circlearrowright S^{2d+2n})$ contains two connected components:

The idea of the proof of Main Theorem

The case where a connected component M of the fixed point set $F(G \circlearrowright \mathbb{C}P^{d+n})$ is homotopically symplectic and not symplectic is covered by the Kaluba-Politarczyk Theorem. In the remaining two cases of M, we argue as follows.

- First, we construct a smooth action of G on S^{2d+2n} such that $F(G \circlearrowright S^{2d+2n})$ contains two connected components: one is the sphere S^{2d}

The idea of the proof of Main Theorem

The case where a connected component M of the fixed point set $F(G \circlearrowright \mathbb{C}P^{d+n})$ is homotopically symplectic and not symplectic is covered by the Kaluba-Politarczyk Theorem. In the remaining two cases of M, we argue as follows.

- First, we construct a smooth action of G on S^{2d+2n} such that $F(G \circlearrowright S^{2d+2n})$ contains two connected components: one is the sphere S^{2d} and the other one is the manifold M.

The idea of the proof of Main Theorem

The case where a connected component M of the fixed point set $F(G \circlearrowright \mathbb{C}P^{d+n})$ is homotopically symplectic and not symplectic is covered by the Kaluba-Politarczyk Theorem. In the remaining two cases of M, we argue as follows.

- First, we construct a smooth action of G on S^{2d+2n} such that $F(G \circlearrowright S^{2d+2n})$ contains two connected components: one is the sphere S^{2d} and the other one is the manifold M. Moreover, at any point $x \in S^{2d}$,

The idea of the proof of Main Theorem

The case where a connected component M of the fixed point set $F(G \circlearrowright \mathbb{C}P^{d+n})$ is homotopically symplectic and not symplectic is covered by the Kaluba-Politarczyk Theorem. In the remaining two cases of M, we argue as follows.

- First, we construct a smooth action of G on S^{2d+2n} such that $F(G \circlearrowright S^{2d+2n})$ contains two connected components: one is the sphere S^{2d} and the other one is the manifold M. Moreover, at any point $x \in S^{2d}$, the normal G-module is

The idea of the proof of Main Theorem

The case where a connected component M of the fixed point set $F(G \circlearrowright \mathbb{C}P^{d+n})$ is homotopically symplectic and not symplectic is covered by the Kaluba-Politarczyk Theorem. In the remaining two cases of M, we argue as follows.

- First, we construct a smooth action of G on S^{2d+2n} such that $F(G \circlearrowright S^{2d+2n})$ contains two connected components: one is the sphere S^{2d} and the other one is the manifold M. Moreover, at any point $x \in S^{2d}$, the normal G-module is the realification of a complex n-dimensional G-module V^n.

The idea of the proof of Main Theorem

The case where a connected component M of the fixed point set $F(G \circlearrowright \mathbb{C}P^{d+n})$ is homotopically symplectic and not symplectic is covered by the Kaluba-Politarczyk Theorem. In the remaining two cases of M, we argue as follows.

- First, we construct a smooth action of G on S^{2d+2n} such that $F(G \circlearrowright S^{2d+2n})$ contains two connected components: one is the sphere S^{2d} and the other one is the manifold M. Moreover, at any point $x \in S^{2d}$, the normal G-module is the realification of a complex n-dimensional G-module V^n.
- Then, we consider the equivariant projectivization

The idea of the proof of Main Theorem

The case where a connected component M of the fixed point set $F(G \circlearrowright \mathbb{C}P^{d+n})$ is homotopically symplectic and not symplectic is covered by the Kaluba-Politarczyk Theorem. In the remaining two cases of M, we argue as follows.

- First, we construct a smooth action of G on S^{2d+2n} such that $F(G \circlearrowright S^{2d+2n})$ contains two connected components: one is the sphere S^{2d} and the other one is the manifold M. Moreover, at any point $x \in S^{2d}$, the normal G-module is the realification of a complex n-dimensional G-module V^n.
- Then, we consider the equivariant projectivization

$$\mathbb{C}P^{d+n} = P(\mathbb{C}^d \oplus \mathbb{C}^1 \oplus V^n) = S(\mathbb{C}^d \oplus \mathbb{C}^1 \oplus V^n)/S^1.$$

The idea of the proof of Main Theorem

The case where a connected component M of the fixed point set $F(G \circlearrowright \mathbb{C}P^{d+n})$ is homotopically symplectic and not symplectic is covered by the Kaluba-Politarczyk Theorem. In the remaining two cases of M, we argue as follows.

- First, we construct a smooth action of G on S^{2d+2n} such that $F(G \circlearrowright S^{2d+2n})$ contains two connected components: one is the sphere S^{2d} and the other one is the manifold M. Moreover, at any point $x \in S^{2d}$, the normal G-module is the realification of a complex n-dimensional G-module V^n.
- Then, we consider the equivariant projectivization

$$\mathbb{C}P^{d+n} = P(\mathbb{C}^d \oplus \mathbb{C}^1 \oplus V^n) = S(\mathbb{C}^d \oplus \mathbb{C}^1 \oplus V^n)/S^1.$$

Hence, $F(G \circlearrowright \mathbb{C}P^{d+n}) \supset \mathbb{C}P^d$ as a connected component

The idea of the proof of Main Theorem

The case where a connected component M of the fixed point set $F(G \circlearrowright \mathbb{C}P^{d+n})$ is homotopically symplectic and not symplectic is covered by the Kaluba-Politarczyk Theorem. In the remaining two cases of M, we argue as follows.

- First, we construct a smooth action of G on S^{2d+2n} such that $F(G \circlearrowright S^{2d+2n})$ contains two connected components: one is the sphere S^{2d} and the other one is the manifold M. Moreover, at any point $x \in S^{2d}$, the normal G-module is the realification of a complex n-dimensional G-module V^n.
- Then, we consider the equivariant projectivization

$$\mathbb{C}P^{d+n} = P(\mathbb{C}^d \oplus \mathbb{C}^1 \oplus V^n) = S(\mathbb{C}^d \oplus \mathbb{C}^1 \oplus V^n)/S^1.$$

  Hence, $F(G \circlearrowright \mathbb{C}P^{d+n}) \supset \mathbb{C}P^d$ as a connected component and V^n occurs as the normal G-module at any $c \in \mathbb{C}P^d$.

The idea of the proof of Main Theorem

The case where a connected component M of the fixed point set $F(G \circlearrowright \mathbb{C}P^{d+n})$ is homotopically symplectic and not symplectic is covered by the Kaluba-Politarczyk Theorem. In the remaining two cases of M, we argue as follows.

- First, we construct a smooth action of G on S^{2d+2n} such that $F(G \circlearrowright S^{2d+2n})$ contains two connected components: one is the sphere S^{2d} and the other one is the manifold M. Moreover, at any point $x \in S^{2d}$, the normal G-module is the realification of a complex n-dimensional G-module V^n.
- Then, we consider the equivariant projectivization

$$\mathbb{C}P^{d+n} = P(\mathbb{C}^d \oplus \mathbb{C}^1 \oplus V^n) = S(\mathbb{C}^d \oplus \mathbb{C}^1 \oplus V^n)/S^1.$$

  Hence, $F(G \circlearrowright \mathbb{C}P^{d+n}) \supset \mathbb{C}P^d$ as a connected component and V^n occurs as the normal G-module at any $c \in \mathbb{C}P^d$.
- Now, we form $\mathbb{C}P^{d+n}{}_c\#_x S^{2d+2n} \cong \mathbb{C}P^{d+n}$

The idea of the proof of Main Theorem

The case where a connected component M of the fixed point set $F(G \circlearrowright \mathbb{C}P^{d+n})$ is homotopically symplectic and not symplectic is covered by the Kaluba-Politarczyk Theorem. In the remaining two cases of M, we argue as follows.

- First, we construct a smooth action of G on S^{2d+2n} such that $F(G \circlearrowright S^{2d+2n})$ contains two connected components: one is the sphere S^{2d} and the other one is the manifold M. Moreover, at any point $x \in S^{2d}$, the normal G-module is the realification of a complex n-dimensional G-module V^n.
- Then, we consider the equivariant projectivization

$$\mathbb{C}P^{d+n} = P(\mathbb{C}^d \oplus \mathbb{C}^1 \oplus V^n) = S(\mathbb{C}^d \oplus \mathbb{C}^1 \oplus V^n)/S^1.$$

  Hence, $F(G \circlearrowright \mathbb{C}P^{d+n}) \supset \mathbb{C}P^d$ as a connected component and V^n occurs as the normal G-module at any $c \in \mathbb{C}P^d$.
- Now, we form $\mathbb{C}P^{d+n}{}_c\#_x S^{2d+2n} \cong \mathbb{C}P^{d+n}$ at two points $c \in \mathbb{C}P^d \subset \mathbb{C}P^{d+n}$ and $x \in S^{2d} \subset S^{2d+2n}$

The idea of the proof of Main Theorem

The case where a connected component M of the fixed point set $F(G \circlearrowright \mathbb{C}P^{d+n})$ is homotopically symplectic and not symplectic is covered by the Kaluba-Politarczyk Theorem. In the remaining two cases of M, we argue as follows.

- First, we construct a smooth action of G on S^{2d+2n} such that $F(G \circlearrowright S^{2d+2n})$ contains two connected components: one is the sphere S^{2d} and the other one is the manifold M. Moreover, at any point $x \in S^{2d}$, the normal G-module is the realification of a complex n-dimensional G-module V^n.
- Then, we consider the equivariant projectivization
$$\mathbb{C}P^{d+n} = P(\mathbb{C}^d \oplus \mathbb{C}^1 \oplus V^n) = S(\mathbb{C}^d \oplus \mathbb{C}^1 \oplus V^n)/S^1.$$
Hence, $F(G \circlearrowright \mathbb{C}P^{d+n}) \supset \mathbb{C}P^d$ as a connected component and V^n occurs as the normal G-module at any $c \in \mathbb{C}P^d$.
- Now, we form $\mathbb{C}P^{d+n}{}_c\#_x S^{2d+2n} \cong \mathbb{C}P^{d+n}$ at two points $c \in \mathbb{C}P^d \subset \mathbb{C}P^{d+n}$ and $x \in S^{2d} \subset S^{2d+2n}$ to obtain that $F(G \circlearrowright \mathbb{C}P^{d+n}) \supset M$ as a connected component.

Doubling of actions

In order to obtain the required smooth action of G on S^{2d+2n},

Doubling of actions

In order to obtain the required smooth action of G on S^{2d+2n}, first we construct a smooth action of G on D^{2d+2n}

Doubling of actions

In order to obtain the required smooth action of G on S^{2d+2n}, first we construct a smooth action of G on D^{2d+2n} such that

$$F(G \circlearrowright D^{2d+2n}) \supset D^{2d} \sqcup M$$

Doubling of actions

In order to obtain the required smooth action of G on S^{2d+2n}, first we construct a smooth action of G on D^{2d+2n} such that

$$F(G \circlearrowright D^{2d+2n}) \supset D^{2d} \sqcup M \quad \text{with} \quad \partial M = \varnothing.$$

Doubling of actions

In order to obtain the required smooth action of G on S^{2d+2n}, first we construct a smooth action of G on D^{2d+2n} such that

$$F(G \circlearrowright D^{2d+2n}) \supset D^{2d} \sqcup M \quad \text{with} \quad \partial M = \varnothing.$$

By forming the G-equivariant double

Doubling of actions

In order to obtain the required smooth action of G on S^{2d+2n}, first we construct a smooth action of G on D^{2d+2n} such that

$$F(G \circlearrowright D^{2d+2n}) \supset D^{2d} \sqcup M \quad \text{with} \quad \partial M = \varnothing.$$

By forming the G-equivariant double

$$\partial(D^{2d+2n} \times D^1) = S^{2d+2n},$$

Doubling of actions

In order to obtain the required smooth action of G on S^{2d+2n}, first we construct a smooth action of G on D^{2d+2n} such that

$$F(G \circlearrowright D^{2d+2n}) \supset D^{2d} \sqcup M \quad \text{with} \quad \partial M = \varnothing.$$

By forming the G-equivariant double

$$\partial(D^{2d+2n} \times D^1) = S^{2d+2n},$$

we obtain a smooth action of G on S^{2d+2n}

Doubling of actions

In order to obtain the required smooth action of G on S^{2d+2n}, first we construct a smooth action of G on D^{2d+2n} such that

$$F(G \circlearrowright D^{2d+2n}) \supset D^{2d} \sqcup M \quad \text{with} \quad \partial M = \varnothing.$$

By forming the G-equivariant double

$$\partial(D^{2d+2n} \times D^1) = S^{2d+2n},$$

we obtain a smooth action of G on S^{2d+2n} such that

$$F(G \circlearrowright S^{2d+2n}) \supset S^{2d} \sqcup M \sqcup M.$$

Doubling of actions

In order to obtain the required smooth action of G on S^{2d+2n}, first we construct a smooth action of G on D^{2d+2n} such that

$$F(G \circlearrowright D^{2d+2n}) \supset D^{2d} \sqcup M \quad \text{with} \quad \partial M = \varnothing.$$

By forming the G-equivariant double

$$\partial(D^{2d+2n} \times D^1) = S^{2d+2n},$$

we obtain a smooth action of G on S^{2d+2n} such that

$$F(G \circlearrowright S^{2d+2n}) \supset S^{2d} \sqcup M \sqcup M.$$

As far as the action of G on D^{2n+2d} is concerned,

Doubling of actions

In order to obtain the required smooth action of G on S^{2d+2n}, first we construct a smooth action of G on D^{2d+2n} such that

$$F(G \circlearrowright D^{2d+2n}) \supset D^{2d} \sqcup M \quad \text{with} \quad \partial M = \varnothing.$$

By forming the G-equivariant double

$$\partial(D^{2d+2n} \times D^1) = S^{2d+2n},$$

we obtain a smooth action of G on S^{2d+2n} such that

$$F(G \circlearrowright S^{2d+2n}) \supset S^{2d} \sqcup M \sqcup M.$$

As far as the action of G on D^{2n+2d} is concerned, it is obtained by the equivariant thickening technique.

Doubling of actions

In order to obtain the required smooth action of G on S^{2d+2n}, first we construct a smooth action of G on D^{2d+2n} such that

$$F(G \circlearrowright D^{2d+2n}) \supset D^{2d} \sqcup M \quad \text{with} \quad \partial M = \varnothing.$$

By forming the G-equivariant double

$$\partial(D^{2d+2n} \times D^1) = S^{2d+2n},$$

we obtain a smooth action of G on S^{2d+2n} such that

$$F(G \circlearrowright S^{2d+2n}) \supset S^{2d} \sqcup M \sqcup M.$$

As far as the action of G on D^{2n+2d} is concerned, it is obtained by the equivariant thickening technique.

The equivariant thickening technique

The equivariant thickening technique

Topology 28 (1989) 273–289

Topology 28 (1989) 273–289

Theorem (K. Pawałowski)

The equivariant thickening technique

Topology 28 (1989) 273–289

Theorem (K. Pawałowski)
Let G be a compact Lie group.

The equivariant thickening technique

Topology 28 (1989) 273–289

Theorem (K. Pawałowski)
Let G be a compact Lie group. Let M be a compact smooth manifold.

The equivariant thickening technique

Topology 28 (1989) 273–289

Theorem (K. Pawałowski)
Let G be a compact Lie group. Let M be a compact smooth manifold. Let X be a finite contractible G-CW complex with $F(G \circlearrowright X) = D^{2d} \sqcup M$.

The equivariant thickening technique

Topology 28 (1989) 273–289

Theorem (K. Pawałowski)
Let G be a compact Lie group. Let M be a compact smooth manifold. Let X be a finite contractible G-CW complex with $F(G \circlearrowright X) = D^{2d} \sqcup M$. Let E be a G-vector bundle over X such that $F(G \circlearrowright E|_{D^{2d} \sqcup M})$

The equivariant thickening technique

Topology 28 (1989) 273–289

Theorem (K. Pawałowski)
Let G be a compact Lie group. Let M be a compact smooth manifold. Let X be a finite contractible G-CW complex with $F(G \circlearrowright X) = D^{2d} \sqcup M$. Let E be a G-vector bundle over X such that $F(G \circlearrowright E|_{D^{2d} \sqcup M})$ is stably isomorphic to $T(D^{2d} \sqcup M)$.

The equivariant thickening technique

Topology 28 (1989) 273–289

Theorem (K. Pawałowski)
Let G be a compact Lie group. Let M be a compact smooth manifold. Let X be a finite contractible G-CW complex with $F(G \circlearrowleft X) = D^{2d} \sqcup M$. Let E be a G-vector bundle over X such that $F(G \circlearrowleft E|_{D^{2d} \sqcup M})$ is stably isomorphic to $T(D^{2d} \sqcup M)$. Then there exists a smooth action

The equivariant thickening technique

Topology 28 (1989) 273–289

Theorem (K. Pawałowski)
Let G be a compact Lie group. Let M be a compact smooth manifold. Let X be a finite contractible G-CW complex with $F(G \circlearrowright X) = D^{2d} \sqcup M$. Let E be a G-vector bundle over X such that $F(G \circlearrowright E|_{D^{2d} \sqcup M})$ is stably isomorphic to $T(D^{2d} \sqcup M)$. Then there exists a smooth action of G on a disk D^{2d+2n} with

$$F(G \circlearrowright D^{2d+2n}) \cong D^{2d} \sqcup M$$

The equivariant thickening technique

Topology 28 (1989) 273–289

Theorem (K. Pawałowski)
Let G be a compact Lie group. Let M be a compact smooth manifold. Let X be a finite contractible G-CW complex with $F(G \circlearrowright X) = D^{2d} \sqcup M$. Let E be a G-vector bundle over X such that $F(G \circlearrowright E|_{D^{2d} \sqcup M})$ is stably isomorphic to $T(D^{2d} \sqcup M)$. Then there exists a smooth action of G on a disk D^{2d+2n} with

$$F(G \circlearrowright D^{2d+2n}) \cong D^{2d} \sqcup M$$

and at any point $x \in D^{2d}$,

The equivariant thickening technique

Topology 28 (1989) 273–289

Theorem (K. Pawałowski)
Let G be a compact Lie group. Let M be a compact smooth manifold. Let X be a finite contractible G-CW complex with $F(G \circlearrowright X) = D^{2d} \sqcup M$. Let E be a G-vector bundle over X such that $F(G \circlearrowright E|_{D^{2d} \sqcup M})$ is stably isomorphic to $T(D^{2d} \sqcup M)$. Then there exists a smooth action of G on a disk D^{2d+2n} with

$$F(G \circlearrowright D^{2d+2n}) \cong D^{2d} \sqcup M$$

and at any point $x \in D^{2d}$, the normal G-module is isomorphic

The equivariant thickening technique

Topology 28 (1989) 273–289

Theorem (K. Pawałowski)
Let G be a compact Lie group. Let M be a compact smooth manifold. Let X be a finite contractible G-CW complex with $F(G \circlearrowright X) = D^{2d} \sqcup M$. Let E be a G-vector bundle over X such that $F(G \circlearrowright E|_{D^{2d} \sqcup M})$ is stably isomorphic to $T(D^{2d} \sqcup M)$. Then there exists a smooth action of G on a disk D^{2d+2n} with

$$F(G \circlearrowright D^{2d+2n}) \cong D^{2d} \sqcup M$$

and at any point $x \in D^{2d}$, the normal G-module is isomorphic to the realification of a complex n-dimensional G-module V^n.

The equivariant thickening technique

Topology 28 (1989) 273–289

Theorem (K. Pawałowski)
Let G be a compact Lie group. Let M be a compact smooth manifold. Let X be a finite contractible G-CW complex with $F(G \circlearrowright X) = D^{2d} \sqcup M$. Let E be a G-vector bundle over X such that $F(G \circlearrowright E|_{D^{2d} \sqcup M})$ is stably isomorphic to $T(D^{2d} \sqcup M)$. Then there exists a smooth action of G on a disk D^{2d+2n} with

$$F(G \circlearrowright D^{2d+2n}) \cong D^{2d} \sqcup M$$

and at any point $x \in D^{2d}$, the normal G-module is isomorphic to the realification of a complex n-dimensional G-module V^n. Also, there exists a G-homotopy equivalence $f \colon D^{2d+2n} \to X$

The equivariant thickening technique

Topology 28 (1989) 273–289

Theorem (K. Pawałowski)
Let G be a compact Lie group. Let M be a compact smooth manifold. Let X be a finite contractible G-CW complex with $F(G \circlearrowright X) = D^{2d} \sqcup M$. Let E be a G-vector bundle over X such that $F(G \circlearrowright E|_{D^{2d} \sqcup M})$ is stably isomorphic to $T(D^{2d} \sqcup M)$. Then there exists a smooth action of G on a disk D^{2d+2n} with

$$F(G \circlearrowright D^{2d+2n}) \cong D^{2d} \sqcup M$$

and at any point $x \in D^{2d}$, the normal G-module is isomorphic to the realification of a complex n-dimensional G-module V^n. Also, there exists a G-homotopy equivalence $f \colon D^{2d+2n} \to X$ such that $f^*(E)$ is stably G-isomorphic to $T(D^{2d+2n})$.

The equivariant thickening technique

Topology 28 (1989) 273–289

Theorem (K. Pawałowski)
Let G be a compact Lie group. Let M be a compact smooth manifold. Let X be a finite contractible G-CW complex with $F(G \circlearrowright X) = D^{2d} \sqcup M$. Let E be a G-vector bundle over X such that $F(G \circlearrowright E|_{D^{2d} \sqcup M})$ is stably isomorphic to $T(D^{2d} \sqcup M)$. Then there exists a smooth action of G on a disk D^{2d+2n} with

$$F(G \circlearrowright D^{2d+2n}) \cong D^{2d} \sqcup M$$

and at any point $x \in D^{2d}$, the normal G-module is isomorphic to the realification of a complex n-dimensional G-module V^n. Also, there exists a G-homotopy equivalence $f\colon D^{2d+2n} \to X$ such that $f^*(E)$ is stably G-isomorphic to $T(D^{2d+2n})$.

Dōmo arigatō gozaimasu!