第３回シューベルトカルキュラスとその周辺

日程：2014 年８月２６日〜８月２９日
場所：〒 700-0005 岡山県岡山市北区理大町１－１
　　　岡山理科大学加計学園５０周年記念館
プログラム委員：成瀬弘（岡山大学）
世話人：池田岳（岡山理科大学)、松村朝雄（KAIST)、中筋麻貴（上智大学）

1 Program

Aug 26, Tuesday

13:30-14:15 T. Ikeda (Okayama University of Scinence)

Overview of the workshop

14:30-15:30 T. Hudson (KAIST, South Korea)

K-theoretic Schubert calculus I

16:00-17:00 D. Sagaki (University of Tsukuba)

Introduction to LS paths I

Aug 27, Wednesday

09:45-10:45 N. Fujita (Tokyo Tech)

A model of Schubert polynomials in terms of Bott-Samelson varieties

11:00-12:00 T. Matsumura (KAIST, South Korea)

K-theoretic Schubert calculus II

13:30-14:30 D. Sagaki (University of Tsukuba)

Introduction to LS paths II

15:00-16:00 C. Li (IPMU)

Equivariant Pieri rule for isotropic Grassmannians

16:15-17:15 M. Nakagawa (Okayama University)

K-homology of affine Grassmannians

Aug 28, Thursday

10:00-11:00 M. Watanabe (University of Tokyo)

On the structure of Schubert modules and filtration by Schubert modules

11:30-12:30 T. Hudson (KAIST, South Korea)

K-theoretic Schubert calculus III

14:15-15:15 L. Mihalcea (Virginia Tech, USA)

An affine deformation of the quantum cohomology ring of flag manifolds and periodic Toda lattice

15:40-16:40 H. Naruse (Okayama University)

Combinatorial K-theory of flag varieties of classical type

17:30 ∼Banquet

Aug 29, Friday

10:00-11:00 M. Nakasuji (Sophia University)

A new approach in Schubert calculus to Casselman's problem

11:30-12:30 T. Matsumura (KAIST, South Korea)

K-theoretic Schubert calculus IV

2 Abstract

Naoki Fujita

A model of Schubert polynomials in terms of Bott-Samelson varieties

The purpose of this expository talk is to explain the results of the paper "Schubert polynomials and Bott-Samelson varieties" (published in 1998) by Peter Magyar. Schubert polynomials are generalizations of Schur polynomials. However, it is not clear how to generalize several classical formulas such as the Weyl character formula, the Demazure character formula, and the generating series of semistandard tableaux. In the paper above, the author introduced a geometric model of Schubert polynomials in terms of Bott-Samelson varieties and obtained these formulas. Here, Bott-Samelson varieties were realized as the orbit closures of a Borel subgroup in a product of Grassmannians. These orbit closures are determined by the Rothe diagrams, and are isomorphic to the usual Bott-Samelson varieties of certain reduced words whose associated chamber families with multiplicity are identical to the Rothe diagrams. In this setting, the generalized formulas above arise naturally from combinatorial arguments about the Rothe diagrams.

Thomas Hudson

Talk 1: On a determinantal formula for the Schubert classes of the Grassmannian

A modern generalization due to Kempf and Laskov of the Giambelli-Thom-Porteous formula allows one to express every Schubert class of the Grassmannian as a determinant in Chern classes. The aim of this talk is to provide a detailed exposition of the proof of the formula, as well as a presentation of the geometric entities and constructions appearing in it: the Grassmannian, its Schubert varieties and their desingularizations.

Talk 2: From the Chow ring to connective K-theory Alternate title: From cohomology to connective K-theory

Connective K-theory is the easiest example of an oriented cohomology theory which encodes information contained in both the Chow ring and in the Grothendieck ring of vector bundles. In this talk I will illustrate how CK^* can be defined starting from algebraic cobordism, present some of its properties and finally analyze how the proof of the determinantal formula has to be modified to suit this more general context.

Takeshi Ikeda

Overview of the workshop

Let us start reviewing some aspects of the Grassmannian and its Schubert subvarieties. How we understand the Schubert conditions, how we can resolve the singularity of the Schubert varieties, and how these considerations lead to the determinantal formula fo the Schubert classes. We will also discuss how the results are related to representation theory. Several talks of this workshop will be variations on theme of the Grassmannian, the prototype of our discussions.

Changzheng Li

Equivariant Pieri rule for isotropic Grassmannians

In this talk, we will introduce an equivariant Pieri rule for Grassmannians of Lie type C, as well as type B and D if time is enough. This is a first manifestly positive formula for isotropic Grassmannians beyond the equivariant Chevalley formula. This is my joint work with Vijay Ravikumar.

Tomoo Matsumura

On Pfaffian formulas for the Schubert classes of the isotropic Grassmannians

In this talk, we consider the Grassmannians of isotropic subspaces in a complex symplectic vector space. Its Schubert classes are first described by Pragacz in terms of Pfaffian in the case of cohomology of Lagrangian Grassmannians. The works of Buch-Kresch-Tamvakis, Kazarian, Ikeda, Wilson, Ikeda-Matsumura, gave its generalization to the equivariant cohomology and also to the non-maximal isotropic case. Recently, in the joint work with Hudson-Ikeda-Naruse, we obtained the analogous Pfaffian (sum) formula for connective K-theory by generalising Kazarian 's method. The aim of this talk is to review the previously existing formulas in cohomology and to explain the detail of the proof of our new formula.

Leonardo Mihalcea

An affine deformation of the quantum cohomology ring of flag manifolds and periodic Toda lattice

A theorem of B. Kim identified the relations of the quantum cohomology ring of the (generalized) flag manifolds with the conserved quantities for the Toda lattice. M. Guest and T. Otofuji, and L. Mare, showed that if a similar quantum cohomology ring exists for affine flag manifolds, then its relations will be determined by the periodic Toda lattice. I will show how to construct a quantum ring which deforms the usual quantum cohomology ring and which depends on an additional affine quantum parameter. It turns out that the conserved quantities of the periodic Toda lattice give the ideal of relations in the new ring. The construction involves a generalization of the notion of "curve neighborhoods" of Schubert varieties, which were defined and studied earlier by the speaker in several joint works with A. Buch, P.E. Chaput, and N. Perrin. The current project is joint with Liviu Mare.

Masaki Nakagawa

K-homology of affine Grassmannian

Let G be a simply-connected simple complex algebraic group and K a maximal compact subgroup of G. The *affine Grassmannian* Gr_G associated to G is defined by $\mathrm{Gr}_G := G(\mathbb{C}((t)))/G(\mathbb{C}[[t]])$. The homology $H_*(\mathrm{Gr}_G)$ and the cohomology $H^*(\mathrm{Gr}_G)$ have remarkable properties because of the following two facts:

1. $H_*(\mathrm{Gr}_G)$ (resp. $H^*(\mathrm{Gr}_G)$) is a free $\mathbb{Z}$-module with a basis consisting of homology (resp. cohomology) *Schubert classes.*
2. It is known that Gr_G is homotopy-equivalent to the based loop group ΩK of K. The group structure of ΩK endows the homology $H_*(\mathrm{Gr}_G)$ and cohomology $H^*(\mathrm{Gr}_G)$ with the structure of dual Hopf algebras over $\mathbb{Z}$.

Therefore one can develop the *Schubert calculus* not only for the cup product in cohomology, but also for the Pontrjagin product in homology. Recently, Lam, Schilling, Shimozono, and Pon have identified these Hopf algebras with certain Hopf algebras of symmetric functions for $G = SL(n, \mathbb{C}), Sp(2n, \mathbb{C})$, and $SO(n, \mathbb{C})$. It is natural to ask for K-theoretic analogues of their results. Especially we are concerned with the K-homology $K_*(\mathrm{Gr}_G)$. At present, only the $SL(n, \mathbb{C})$ case has been established by Lam, Schilling, and Shimozono. In this talk, I will explain the current status of our research for the case $G = Sp(2n, \mathbb{C})$ and $SO(n, \mathbb{C})$. This is joint work with H. Naruse.

Maki Nakasuji

A new approach in Schubert calculus to Casselman's problem

Casselman basis is the basis of the vectors in a spherical representation of a reductive p-adic group that are fixed by the Iwahori subgroup. Casselman 's problem is to express it in terms of the natural basis of Iwahori-fixed vectors. In this talk, we will discuss this problem from the view point of Schubert calculus. This will be a report on a joint work in progress with H.Naruse. More precisely, we will see our new approach compared with that by Brubaker-Bump-Licata using the Demazure-Lusztig operators.

Hiroshi Naruse

Combinatorial K-theory of flag varieties of classical type

We survey the combinatorial aspects of K-theory of flag varieties of classical type by using Yang-Baxter relations and excited Young diagrams.

Daisuke Sagaki

Introduction to LS paths I-II

I'd like to give survey lectures on Littelmann's path models: In the papers "A Littlewood-Richardson rule for symmetrizable Kac-Moody algebras (1994)" and "Paths and root operators in representation theory (1995)", Littelmann introduced Lakshmibai-Seshadri (LS) paths of shape λ (where λ is an integral weight), and gave the set of them a crystal structure in terms of his root operators; if λ is dominant, then LS paths of shape λ are defined in terms of the Bruhat order (or, Bruhat graph), and the crystal of LS paths of shape λ is isomorphic to the crystal basis of the integrable highest weight module of highest weight λ (proved independently by Kashiwara and Joseph). Littelmann also gave a characterization of the Demazure subcrystal (which is contained in the crystal basis of an integrable highest weight module) in terms of the initial directions of LS paths. If I have time, I'll also explain quantum LS paths and semi-infinite LS paths, which were introduced in my joint work with Lenart, Naito, Schilling, Shimozono, and joint work with Ishii and Naito, respectively. Quantum (resp., semi-infinite) LS paths are defined in terms of the quantum (resp., semi-infinite) Bruhat graph, instead of the ordinary Bruhat graph, and the crystal of them is isomorphic to the crystal basis of a Weyl module, i.e., a tensor product of level-zero fundamental modules (resp., an extremal weight module).

Masaki Watanabe

On the structure of Schubert modules and filtration by Schubert modules

Schubert modules, introduced by Kraskiewicz and Pragacz, are certain representations of the group of all upper triangular matrices. They have the property that their characters are just (single) Schubert polynomials, and thus the class of modules having a filtration by Schubert modules is deeply related with the study of Schubert-posivitity. I will talk about my recent results on such class of modules.