K-HOMOLOGY OF AFFINE GRASSMANNIANS

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ABSTRACT. Let G be a simply-connected simple complex algebraic group and K a maximal compact subgroup of G. The *affine Grassmannian* Gr_G associated to G is defined by $\mathrm{Gr}_G := G(\mathbb{C}((t)))/G(\mathbb{C}[[t]])$. The homology $H_*(\mathrm{Gr}_G)$ and the cohomology $H^*(\mathrm{Gr}_G)$ have remarkable properties because of the following two facts:

(1) $H_*(\mathrm{Gr}_G)$ (resp. $H^*(\mathrm{Gr}_G)$) is a free $\mathbb{Z}$-module with a basis consisting of homology (resp. cohomology) *Schubert classes*.

(2) It is known that Gr_G is homotopy-equivalent to the based loop group ΩK of K. The group structure of ΩK endows the homology $H_*(\mathrm{Gr}_G)$ and cohomology $H^*(\mathrm{Gr}_G)$ with the structure of dual Hopf algebras over $\mathbb{Z}$.

Therefore one can develop the *Schubert calculus* not only for the cup product in cohomology, but also for the Pontrjagin product in homology. Recently, Lam, Schilling, Shimozono, and Pon have identified these Hopf algebras with certain Hopf algebras of symmetric functions for $G = SL(n, \mathbb{C}), Sp(2n, \mathbb{C})$, and $SO(n, \mathbb{C})$. It is natural to ask for K-theoretic analogues of their results. Especially we are concerned with the K-homology $K_*(\mathrm{Gr}_G)$. At present, only the $SL(n, \mathbb{C})$ case has been established by Lam, Schilling, and Shimozono. In this talk, I will explain the current status of our research for the case $G = Sp(2n, \mathbb{C})$ and $SO(n, \mathbb{C})$. This is joint work with H. Naruse.

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